Where Quants Go Wrong

A dozen basic lessons in commonsense for quants and risk managers and the traders who rely on them

Paul Wilmott

Paul Wilmott is a financial consultant, specializing in derivatives, risk management and quantitative finance. Paul is the author of *Paul Wilmott Introduces Quantitative Finance* (Wiley 2007), *Paul Wilmott On Quantitative Finance* (Wiley 2006), *Frequently Asked Questions in Quantitative Finance* (Wiley 2006) and other financial textbooks. He has written over 100 research articles on finance and mathematics.

Paul Wilmott was a founding partner of the volatility arbitrage hedge fund Caissa Capital which managed $170million. His responsibilities included forecasting, derivatives pricing, and risk management.

Dr Wilmott is the proprietor of www.wilmott.com, the popular quantitative finance community website, the quant magazine *Wilmott* and is the Course Director for the Certificate in Quantitative Finance www.7city.com/cqf. The CQF teaches how to apply mathematics to finance, focusing on implementation and pragmatism, robustness and transparency.

Quants and risk managers keep making the same mistakes over and over again. Each time this happens the losses increase in value. A small amount of commonsense and some basic mathematics can stop this happening. However, this requires quants, risk managers and their bosses to move their attention away from complexity towards robustness of models.*

Keywords: Jensen's inequality; Sensitivity to parameters; Correlation; Diversification; Dynamic hedging; Feedback; Supply and demand; Closed-form solutions; Calibration; Modelling; Precision; Nonlinearity; CSI Miami

*This paper was borne out of a CQF lecture (www.7city.com/cqf) and a series of blogs (www.wilmott.com/blogs/paul).

Topics

- Lack of diversification
- Supply and demand
- Jensen's inequality arbitrage
- Sensitivity to parameters
- Correlation
- Reliance on continuous hedging (arguments)
- Feedback
- Reliance on closed-form solutions
- Valuation is not linear
- Calibration
- Too much precision
- Too much complexity

But first a simple test-yourself quiz.

Quiz

Question 1: What are the advantages of diversification among products, or even among mathematical models?

Question 2: If you add risk and curvature what do you get?

Question 3: If you increase volatility what happens to the value of an option?

Question 4: If you use ten different volatility models to value an option and they all give you very similar values what can you say about volatility risk?

Question 5: One apple costs 50p, how much will 100 apples cost you?

Lesson 1: Lack of Diversification

Example: It's your first day as a trader in a bank. You're fresh out of an Ivy League Masters program. You're keen and eager, you want to do the best you can in your new job, you want to impress your employer and make your family proud. So, what do you trade? What strategies should you adopt? Having been well educated in theoretical finance you know that it's important to diversify, that by diversifying you can increase expected return and decrease your risk. Let's put that into practice.

- Traders coin tossing

- Banks copying each other

Keynes said, “It is better to fail conventionally than to succeed unconventionally.”

There is no incentive to diversify while you are playing with OPM (Other People’s Money).

Example: Exactly the same as above but replace ‘trades’ with ‘models.’ There is also no incentive to use different models from everyone else, even if your are better.

There's a timescale issue here as well. Anyone can sell deep OTM puts for far less than any 'theoretical' value, not hedge them, and make a fortune for a bank, which then turns into a big bonus for the individual trader. You just need to be able to justify this using some convincing model. Eventually things will go pear shaped and you'll blow up. However, in the meantime everyone jumps onto the same (temporarily) profitable bandwagon, and everyone is getting a tidy bonus. The moving away from unprofitable trades and models seems to be happening slower than the speed at which people are accumulating bonuses from said trades and models!

Envy:

We all know of behavioural finance experiments such as the following two questions.

First question, people are asked to choose which world they would like to be in, all other things being equal, World A or World B where

A. You have 2 weeks' vacation, everyone else has 1 week

B. You have 4 weeks' vacation, everyone else has 8 weeks

The large majority of people choose to inhabit World B. They prefer more holiday to less in an absolute sense, they do not suffer from vacation envy.

But then the second question is to choose between World A and World B in which

A. You earn \$50,000 per year, others earn \$25,000 on average

B. You earn \$100,000 per year, others earn \$200,000 on average

Goods have the same values in the two worlds. Now most people choose World A, even though you won't be able to buy as much 'stuff' as in World B. But at least you'll have more 'stuff' than your neighbours. People suffer a great deal from financial envy.

In banking the consequences are that people feel the need to do the same as everyone else, for fear of being left behind. Again, diversification is just not in human nature.

Lesson 2: Supply and Demand

Supply and demand is what ultimately drives everything! But where is the supply and demand parameter or variable in Black–Scholes?

A trivial observation: The world is net long equities after you add up all positions and options. So, net, people worry about falling markets. Therefore people will happily pay a premium for out-of-the-money puts for downside protection. The result is that put prices rise and you get a negative skew. That skew contains information about demand and supply and *not* about the only 'free' parameter in Black–Scholes, the volatility.

The complete-market assumption is obviously unrealistic, and importantly it leads to models in which a small number of parameters are used to capture a large number of effects.

The price of milk is a scalar quantity that has to capture in a single number all the behind-the-scenes effects of, yes, production, but also supply and demand, salesmanship, etc. Perhaps the pint of milk is even a 'loss leader.' A vector of inputs produces a scalar price.

So, no, you cannot back out the cost of production from a single price.

Similarly

- you cannot back out a precise volatility from the price of an option when that price is also governed by supply and demand, fear and greed, not to mention all the imperfections that mess up your nice model (hedging errors, transaction costs, feedback effects, etc.).

Supply and demand dictate prices, assumptions and models impose constraints on the relative prices among instruments. Those constraints can be strong or weak depending on the strength or weakness of the assumptions and models.

Lesson 3: Jensen's Inequality Arbitrage

Jensen's Inequality states that if $f(\cdot)$ is a convex function and x is a random variable then

$$E\left[f(x)\right] \geq f\left(E[x]\right).$$

This justifies why non-linear instruments, options, have inherent value.

Example: You roll a die, square the number of spots you get, and you win that many dollars. How much is this game worth? (Assuming you expect to break even.) We know that the average number of spots on a fair die is $3\frac{1}{2}$ but the fair 'price' for this bet is not $\left(3\frac{1}{2}\right)^2$.

For this exercise $f(x)$ is x^2, it is a convex function. So

$$E[x] = 3\tfrac{1}{2}$$

and

$$f\left(E[x]\right) = \left(3\tfrac{1}{2}\right)^2 = 12\tfrac{1}{4}.$$

But

$$E\left[f(x)\right] = \frac{1+4+9+16+25+36}{6} = 15\tfrac{1}{6} > f\left(E[x]\right).$$

The fair price is $15\frac{1}{6}$.

Jensen's inequality and convexity can be used to explain the relationship between randomness in stock prices and the value inherent in options, the latter typically having some convexity.

Suppose that a stock price S is random and we want to consider the value of an option with payoff $P(S)$.

If the payoff is convex then

$$E[P(S_T)] \geq P(E[S_T]).$$

We can get an idea of how much greater the left-hand side is than the right-hand side by using a Taylor series approximation around the mean of S. Write

$$S = \bar{S} + \epsilon,$$

where $\bar{S} = E[S]$, so $E[\epsilon] = 0$. Then

$$E\left[f(S)\right] = E\left[f(\bar{S}+\epsilon)\right] = E\left[f(\bar{S}) + \epsilon f'(\bar{S}) + \tfrac{1}{2}\epsilon^2 f''(\bar{S}) + \cdots\right]$$

$$\approx f(\bar{S}) + \tfrac{1}{2}f''(\bar{S})E\left[\epsilon^2\right]$$

$$= f(E[S]) + \tfrac{1}{2}f''(E[S])E\left[\epsilon^2\right].$$

Therefore the left-hand side is greater than the right by approximately

$$\tfrac{1}{2}f''(E[S])\,E\left[\epsilon^2\right].$$

This shows the importance of two concepts

- $f''(E[S])$: This is the **convexity** of an option. As a rule this adds value to an option. It also means that any intuition we may get from linear contracts (forwards and futures) might not be helpful with non-linear instruments such as options.

- $E\left[\epsilon^2\right]$: This is the **variance** of the return on the random underlying. Modelling randomness is the key to valuing options.

The lesson to learn from this is that whenever a contract has convexity in a variable or parameter, and that variable or parameter is random, then allowance must be made for this in the pricing.

Example: Anything depending on forward rates. If you price a fixed-income instrument with the assumption that forward rates are fixed (the deterministic models of yield, duration, etc.) and there is some nonlinearity in those rates then you are missing value. How much value depends on the convexity with respect to the forward rates and forward rate volatility.*

Example: Some things are tricky to model and so one tends to assume they are deterministic. Mortgage-backed securities have payoffs, and therefore values, that depend on prepayment. Often one assumes prepayment to be a deterministic function of interest rates, this can be dangerous. Try to quantify the convexity with respect to prepayment and the variance of prepayment.

*By 'convexity with respect to forward rates' I do not mean the curvature in the forward rate curve, I mean the second derivative of the contract with respect to the rates.

Lesson 4: Sensitivity To Parameters

If volatility goes up what happens to the value of an option? Did you say the value goes up? Oh dear, bottom of the class for you! I didn't ask what happens to the value of a vanilla option, I just said "an" option, of unspecified terms.

- Pricing an up-and-out call option
- Early-morning panic
- Vega
- Getting fired

What went wrong was that you assumed volatility to be constant in the option formula/model and then you changed that constant. This is only valid if you know that the parameter is constant but are not sure what that constant is. But that's not a realistic scenario in finance. In fact, I can only think of one scenario where this makes sense. . .

- A hot tip from God

By varying a constant parameter you are effectively measuring

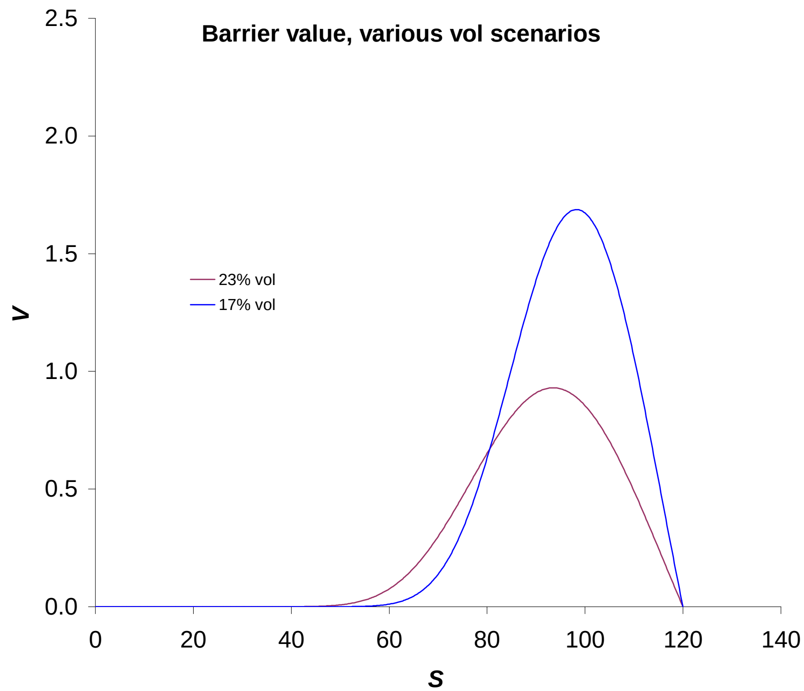
$$\frac{\partial V}{\partial \text{ parameter}}.$$

This is what you are doing when you measure the ‘greek’ vega:

$$\text{vega } = \frac{\partial V}{\partial \sigma}.$$

But this greek is misleading. Those greeks which measure sensitivity to a ‘variable’ are fine, those which supposedly measure sensitivity to a ‘parameter’ are not. Plugging different constants for volatility over the range 17% to 23% is *not* the same as examining the sensitivity to volatility when it is allowed to roam freely between 17 and 23% *without the constraint of being constant*. I call such greeks “bastard greeks” because they are illegitimate.

An example:



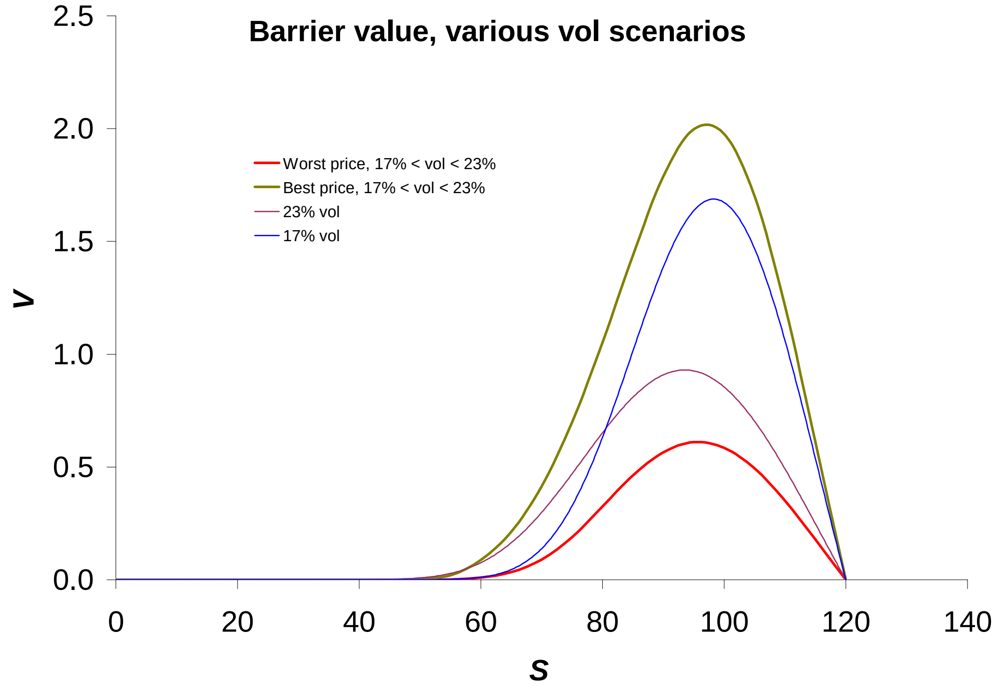
The value of some up-and-out call option using volatilities 17% and 23%.

The problem arises because this option has a gamma that changes sign.

The relationship between sensitivity to volatility and gamma is because they always go together. In the Black–Scholes equation we have a term of the form

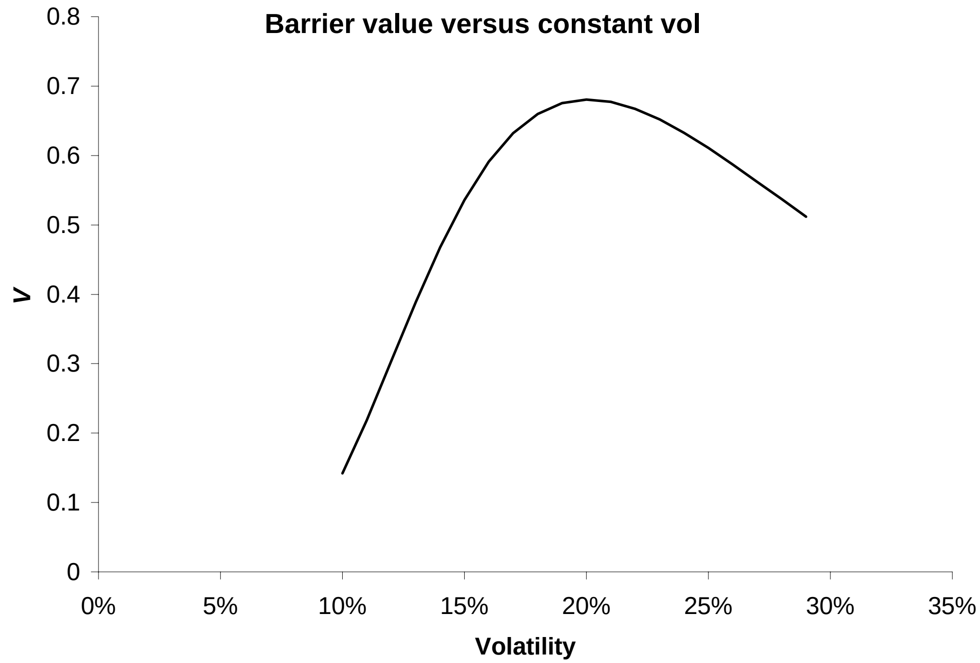
$$\frac{1}{2}\sigma_2 S^2 \frac{\partial^2 V}{\partial S^2}.$$

The bigger this combined term is, the more the option is worth. But if gamma is negative large volatility makes this big in absolute value, but negative, so it decreases the option's value.



Uncertain volatility model, best and worst cases.

Meaningless ness of implied volatility:

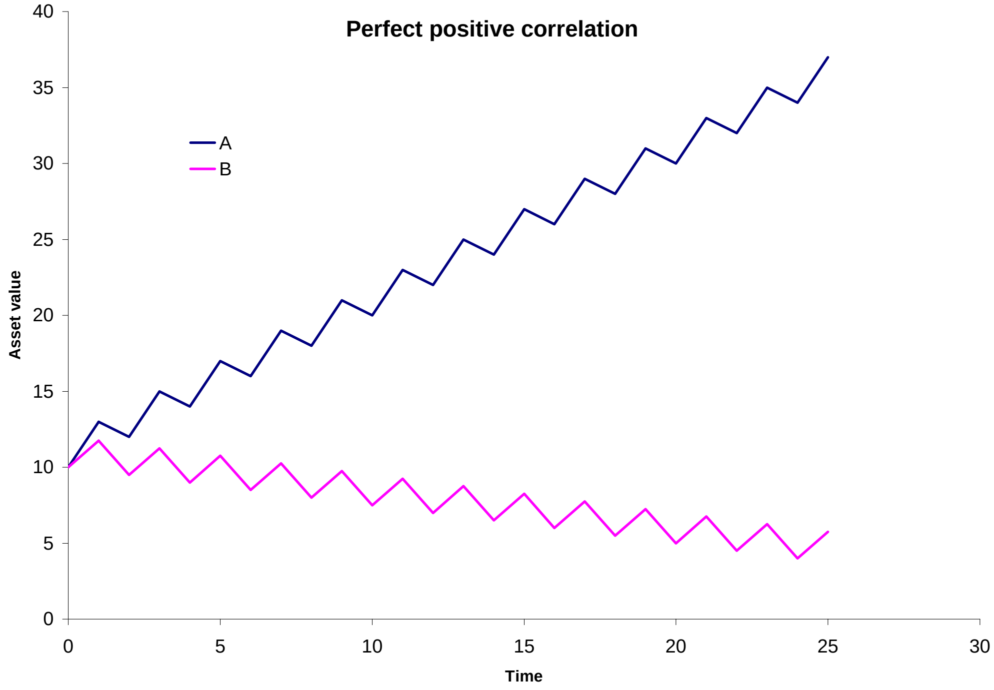


Value versus constant volatility.

Lesson 5: Correlation

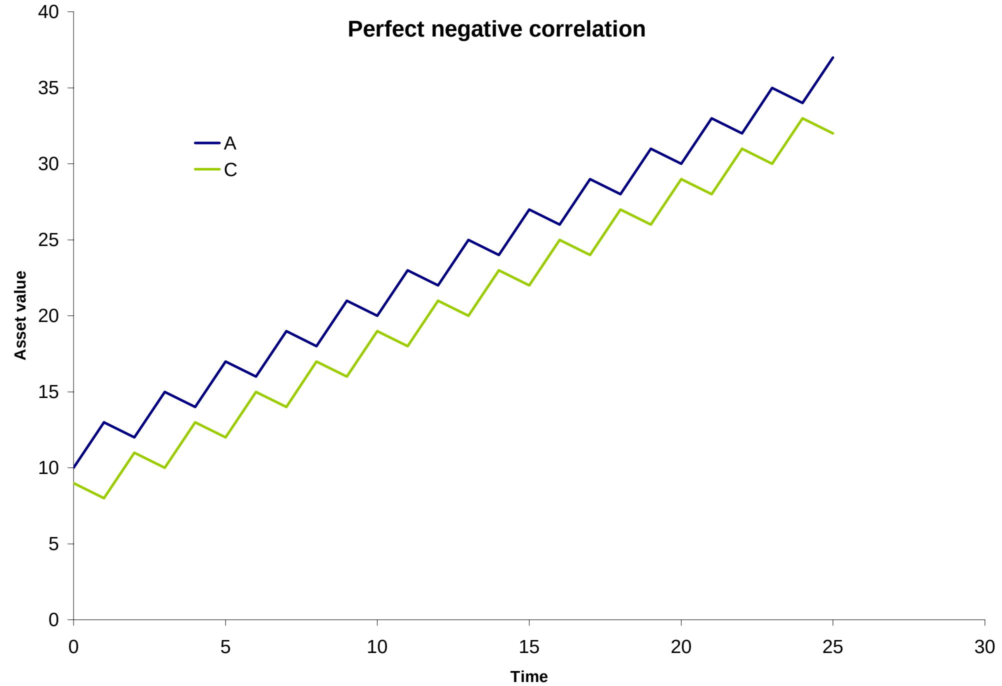
- Quant toolbox

When we think of two assets that are highly correlated then we are tempted to think of them both moving along side by side almost. Surely if one is growing rapidly then so must the other? This is not true.



Two perfectly correlated assets.

And if two assets are highly negatively correlated then they go in opposite directions? No, again not true.



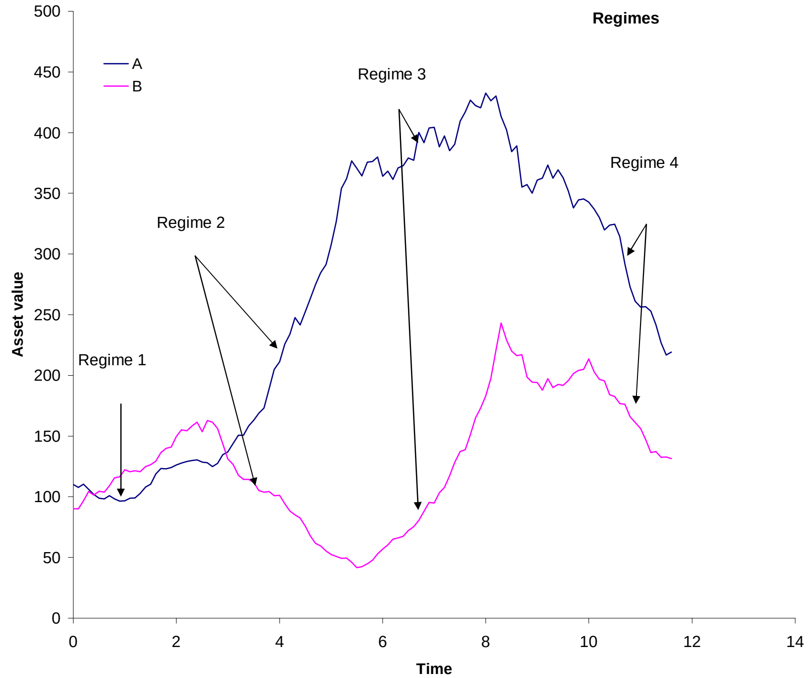
Two perfectly negatively correlated assets.

If we are modelling using stochastic differential equations then correlation is about what happens at the smallest, technically infinitesimal, timescale. It is not about the 'big picture' direction. This can be very important and confusing. For example, if we are interested in how assets behave over some finite time horizon then we still use correlation even though we typically don't care about short timescales only our longer investment horizon (at least in theory).

However, if we are hedging an option that depends on two or more underlying assets then, conversely, we don't care about direction (because we are hedging), only about dynamics over the hedging timescale. The use of correlation may then be easier to justify. But then we have to ask how stable is this correlation.

So when wondering whether correlation is meaningful in any problem you must answer two questions (at least), one concerning timescales (investment horizons or hedging period) and another about stability.

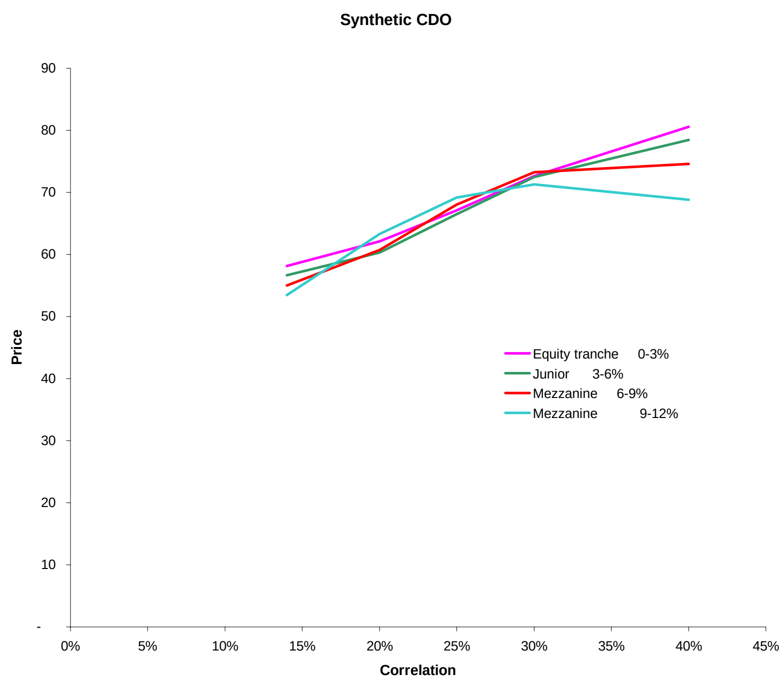
- Running shoes



Two assets, four regimes.

As you can see, the dynamics between just two companies can be fascinating. And can be modelled using all sorts of interesting mathematics. One thing is for sure and that is such dynamics while fascinating are certainly not captured by a correlation of 0.6!

Example: Synthetic CDOs suffer from problems with correlation. People typically model these using a copula approach, and then argue about which copula to use. Finally because there are so many parameters in the problem they say "Let's assume they are all the same!" Then they vary that single constant correlation to look for sensitivity (and to back out implied correlations). Where do I begin criticizing this model? Let's say that just about everything in this model is stupid and dangerous. The model does not capture the true nature of the interaction between underlyings, correlation never does, and then making such an enormously simplifying assumption about correlation is just bizarre. (I grant you not as bizarre as the people who lap this up without asking any questions.)



Various tranches versus correlation.

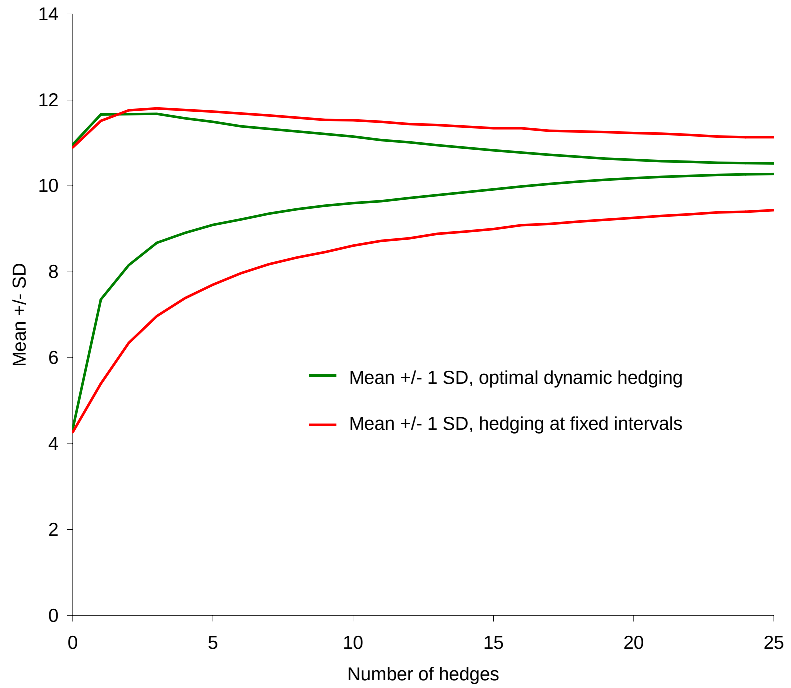
Lesson 6: Reliance on Continuous Hedging (Arguments)

One of the most important concepts in quantitative finance is that of delta or dynamic hedging. This is the idea that you can hedge risk in an option by buying and selling the underlying asset. This is called delta hedging since 'delta' is the Greek letter used to represent the amount of the asset you should sell. Classical theories require you to rebalance this hedge continuously. In some of these theories, and certainly in all the most popular, this hedging will perfectly eliminate all risk. Once you've got rid of risk from your portfolio it is easy to value since it should then get the same rate of return as putting money in the bank.

This is a beautiful, elegant, compact theory, with lots of important consequences. Two of the most important consequences (as well as the most important which is...no risk!) are that, first, only volatility matters in option valuation, the direction of the asset doesn't, and, second, if two people agree on the level of volatility they will agree on the value of an option, personal preferences are not relevant.

The assumption of continuous hedging seems to be crucial to this theory. But is this assumption valid?

The figure shows a comparison between the values of an at-the-money call, strike 100, one year to expiration, 20% volatility, 5% interest rate, when hedged at fixed intervals (the red line) and hedged optimally (the green line). The lines are the mean value plus and minus one standard deviation. All curves converge to the Black-Scholes complete-market, risk-neutral, price of 10.45, but hedging optimally gets you there much faster. If you hedge optimally you will get as much risk reduction from just 10 rehedges as if you use 25 equally spaced rehedges.



Risk reduction when hedging discretely.

From this we can conclude that as long as people know the best way to dynamically hedge then we may be able to get away with using risk neutrality even though hedging is not continuous.

But do they know this?

Everyone is brought up on the results of continuous hedging, and they rely on them all the time, but they almost certainly do not have the necessary ability to make those results valid!

Lesson 7: Feedback

Are derivatives a good thing or a bad thing? Their origins are in hedging risk, allowing producers to hedge away financial risk so they can get on with making pork bellies or whatever. Now derivatives are used for speculation, and the purchase/sale of derivatives for speculation outweighs their use for hedging.

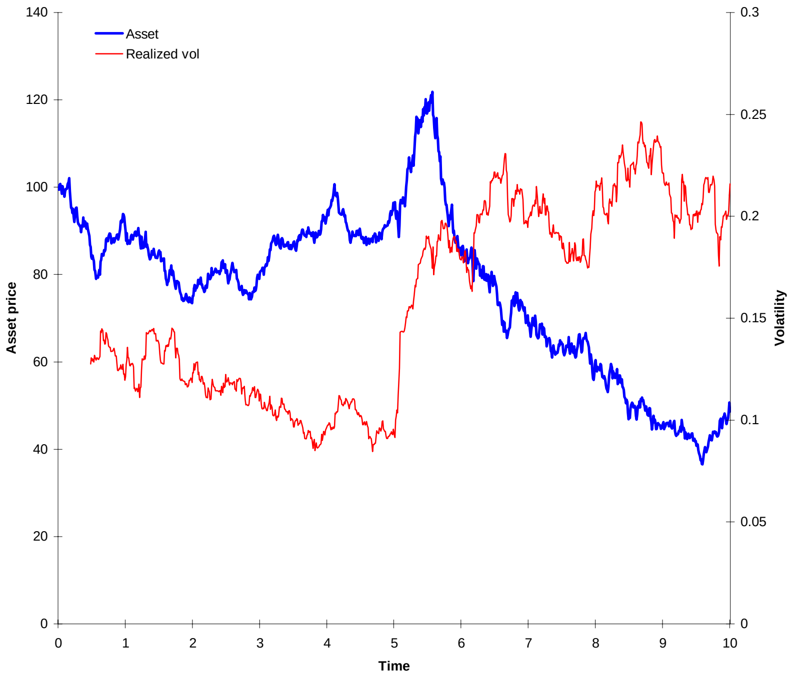
Does this matter? We know that speculation with linear forwards and futures can affect the spot prices of commodities, especially in those that cannot easily be stored. But what about all the new-fangled derivatives that are out there?

A simplistic analysis would suggest that derivatives are harmless, since for every long position there is an equal and opposite short position, and they cancel each other out. But this misses the important point that usually one side or the other is involved in some form of dynamic hedging used to reduce their risk. Often one side buys a contract so as to speculate on direction of the underlying. The seller is probably not going to have exactly the opposite view on the market and so they must hedge away risk by dynamically hedging with the underlying. And that dynamic hedging using the underlying can move the market. This is the tail wagging the dog!

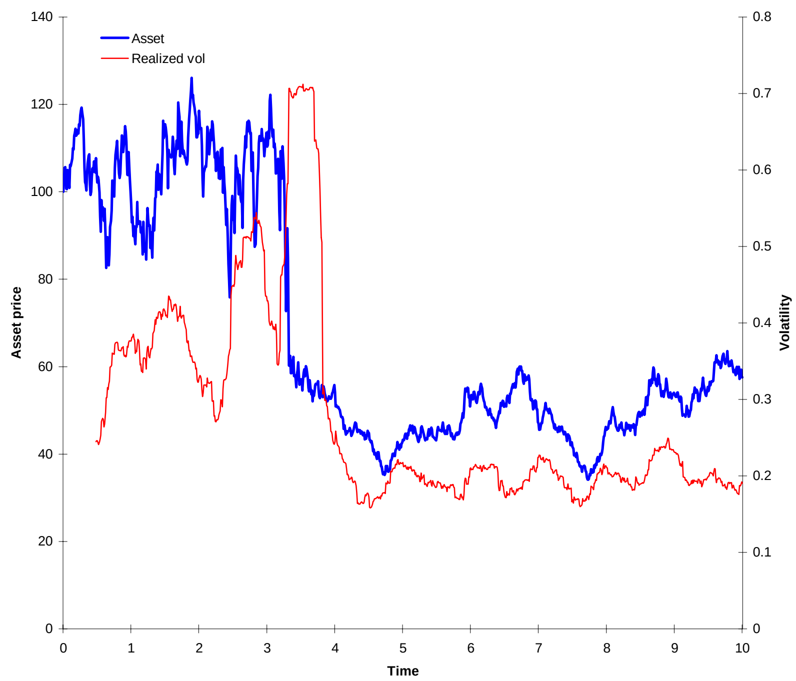
There are two famous examples of this feedback effect:

- Convertible bonds—volatility decrease

- 1987 crash and (dynamic) portfolio insurance—volatility increase



Simulation when hedging long gamma.



Simulation when hedging short gamma.

Lesson 8: Reliance on Closed-form Solutions

Example: You need to value a fixed-income contract and so you have to choose a model. Do you (a) analyze historical fixed-income data in order to develop an accurate model, this is then solved numerically, and finally back tested using a decade's worth of past trades to test for robustness, or (b) use Professor X's model because the formulæ are simple and, quite frankly, you don't know any numerical analysis, or (c) do whatever everyone else is doing? Typically people will go for (c), partly for reasons already discussed, which amounts to (b).

Example: You are an aeronautical engineer designing a new airplane. Boy, those Navier–Stokes equations are hard! How do you solve non-linear equations? Let's simplify things, after all you made a paper plane as a child, so let's just scale things up. The plane is built, a big engine is put on the front, it's filled with hundreds of passengers, and it starts its journey along the runway. You turn your back, without a thought for what happens next, and start on your next project.

One of those examples is fortunately not real. Unfortunately, the other is.

Quants love closed-form solutions. The reasons are

1. Pricing is faster

2. Calibration is easier

3. You don't have to solve numerically

Popular examples of closed-form solutions/models are, in equity derivatives, the Heston stochastic volatility model, and in fixed income, Vasicek,* Hull & White, etc.

Models with closed-form solutions have several roles in quantitative finance. Closed-form solutions are

- useful for preliminary insight

- good for testing your numerical scheme before going on to solve the real problem

- for examining second-year undergraduate mathematicians

*To be fair to Vasieck I'm not sure he ever claimed he had a great model, his paper set out the general theory behind the spot-rate models, with what is now known as the Vasicek model just being an example.

Lesson 9: Valuation is Not Linear

You want to buy an apple, so you pop into Waitrose. An apple will cost you 50p. Then you remember you've got friends coming around that night and these friends really adore apples. Maybe you should buy some more? How much will 100 apples cost?

Here's a quote from a well-known book: "The change of numeraire technique probably seems mysterious. Even though one may agree that it works after following the steps in the chapter, there is probably a lingering question about why it works. The author's opinion is that it may be best simply to regard it as a 'computational trick'. Fundamentally it works *because valuation is linear*. . . . The linearity is manifested in the statement that the value of a cash flow is the sum across states of the world of the state prices multiplied by the size of the cash flow in each state. . . . After enough practice with it, it will seem as natural as other computational tricks one might have learned."

Note it doesn't say that linearity is an assumption, it is casually taken as a fact. Valuation is apparently linear. Now there's someone who has never bought more than a single apple!

Example: The same author may be on a sliding royalty scale so that the more books he sells the bigger his percentage. How can nonlinearity be a feature of something as simple as buying apples or book royalties yet not be seen in supposedly more complex financial structured products?

Example: A bank makes a million dollars profit on CDOs. Fantastic! Let's trade 10 times as much! They make $10million profit. The bank next door hears about this and decides it wants a piece of the action. They trade the same size. Between the two banks they make $18million profit. Where'd the $2million go? Competition between them brings the price and profit margin down. To make up the shortfall, and because of simple greed, they increase the size of the trades. Word spreads and more banks join in. Profit margins are squeezed. Total profit stops rising even though the positions are getting bigger and bigger. And then the inevitable happens, the errors in the models exceed the profit margin (margin for error), and between them the banks lose billions. "Fundamentally it works because valuation is linear." Oh dear!

To appreciate the importance of nonlinearity you have to understand that there is a difference between the value of a portfolio of contracts and the sum of the values of the individual contracts.* In pseudomath, if the problem has been set up properly, you will get

$$\text{Value}(A+B) \geq \text{Value}(A) + \text{Value}(B).$$

*What I call, to help people remember it, the 'Beatles effect.' The Fab Four being immeasurably more valuable as a group than as individuals. . . Wings, Thomas the Tank Engine,. . .

- Static hedging of a barrier option

Here is a partial list of the advantages to be found in some non-linear models.

- Perfect calibration
- Speed
- Easy to add complexity to the model
- Optimal static hedging
- Can be used by buy and sell sides

Reading list:

- Hoggard, Whalley & Wilmott (1994) on costs

- Avellaneda, Levy & Parás (1995) on the Uncertain Volatility Model (UVM)

- In Hua & Wilmott on modelling crashes and CrashMetrics

- In Ahn & Wilmott on mean-variance pricing and hedging

Lesson 10: Calibration

"A cynic is a man who knows the price of everything and the value of nothing," said Oscar Wilde.

Example: Wheels cost \$10 each. A soapbox is \$20. How much is a go-cart? The *value* is \$60.

Price?

Worth?

- The WWVN

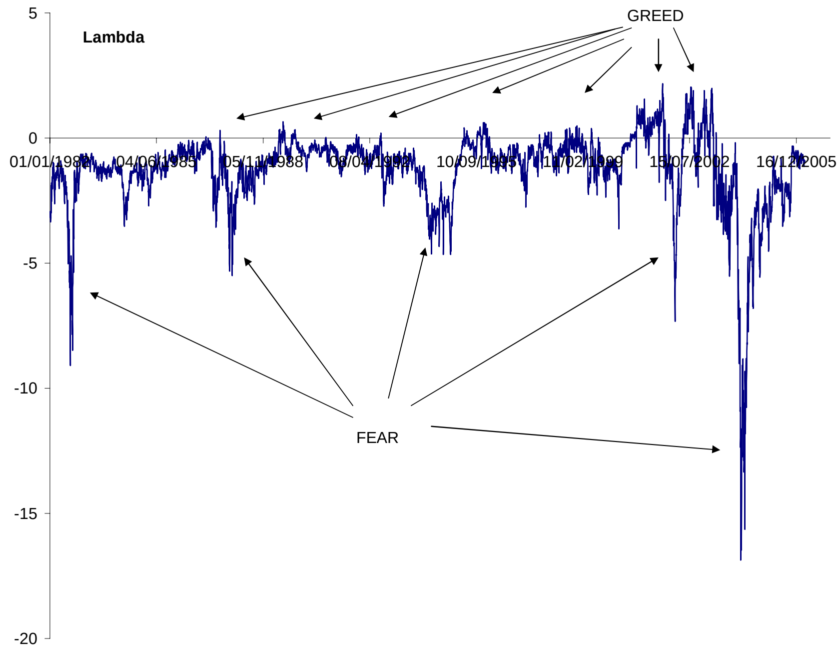
- inverse problems
- frozen fish
- CSI Miami
- crystal balls

Problems with calibration:

- Over fitting. You lose important predictive information if your model fits perfectly. The more instruments you calibrate to the less use that model is.
- Fudging hides model errors: Perfect calibration makes you think you have no model risk, when in fact you probably have more than if you hadn't calibrated at all.
- Always unstable. The parameters or surfaces always change when you recalibrate.
- Confusion between actual parameter values and those seen via derivatives. For example there are two types of credit risk, the actual risk of default and the market's perceived risk of default.

Why is calibration unstable?

- Market Price of Risk



Market price of interest rate risk versus time.

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When you calibrate you are saying that whatever the market sentiment is today, as seen in option prices, is going to pertain forever. So if the market is panicking today it will always panic. But the figure shows that such extremes of emotion are short-lived. And so if you come back a week later you will now be calibrating to a market that has ceased panicking and is perhaps now greedy!

Calibration assumes a structure for the future that is inconsistent with experience, inconsistent with common sense, and that fails all tests.

Lesson 11: Too much precision

Given all the errors in the models, their unrealistic assumptions, and the frankly bizarre ways in which they are used, it is surprising that banks and funds make money at all!

- Zero sum

- Who owns the world

1. There is demand for some contract, real or perceived

2. The contract must be understood in terms of risk, valuation, potential market, profit, etc.

3. A deal gets done with an inbuilt profit margin

4. The contract is then thrown into some big pot with lots of other contracts and they are risk managed together

5. A profit is accrued, perhaps marking to model, or perhaps at expiration

Stages 2 and 4 are inconsistent.

The point of this lesson is to suggest that more effort is spent on the benefits of portfolios than on fiddly niceties of modelling to an obsessive degree of accuracy. Accept right from the start that the modelling is going to be less than perfect. It is not true that one makes money from every instrument because of the accuracy of the model. Rather one makes money *on average* across *all* instruments *despite* the model. These observations suggest to me that less time should be spent on dodgy models, meaninglessly calibrated, but more time on models that are accurate *enough* and that build in the benefits of portfolios.

Some models are better than others. Sometimes even working with not-so-good models is not too bad. To a large extent what determines the success of models is the type of market. Let me give some examples.

Equity, FX and commodity markets: Here the models are only so-so. There has been a great deal of research on improving these models, although not necessarily productive work. Combine less-than-brilliant models with potentially very volatile markets and exotic, non-transparent, products and the result can be dangerous. On the positive side as long as you diversify across instruments and don't put all your money into one basket then you should be ok.

Fixed-income markets: These models are pretty dire. So you might expect to lose (or make) lots of money. Well, it's not as simple as that. There are two features of these markets which make the dire modelling less important, these are a) the underlying rates are not very volatile and b) there are plenty of highly liquid vanilla instruments with which to try to hedge model risk. (I say "try to" because most model-risk hedging is really a fudge, inconsistent with the framework in which it is being used.)

Correlation markets: Oh, Lord! Instruments whose pricing requires input of correlation (FI excepted, see above) are accidents waiting to happen. The dynamic relationship between just two equities can be beautifully complex, and certainly never to be captured by a single number, correlation. Fortunately these instruments tend not to be bought or sold in non-diversified, bank-destroying quantities. (Except for CDOs, of course!)

Credit markets: Single-name instruments are not too bad. Again problems arise with any instrument that has multiple 'underlyings,' so the credit derivatives based on baskets. . . you know who you are. But as always, as long as the trades aren't too big then it's not the end of the world.

Lesson 12: Too Much Complexity

"Four stochastic parameters good, two stochastic parameters bad." (Thanks to George Orwell.)

- Selling books to non mathematicians

- Impressing people (books with yellow covers)

- 30 pages or 4 pages (Hyungsok Ahn)

- Rule 1 of quant finance seems to be 'Make this as difficult as we can.'

- Brainteaser on wilmott.com: Girsanov, Doleans-Dade martingales, and optimal stopping

Bonus Lesson 13: The Binomial Method is Rubbish

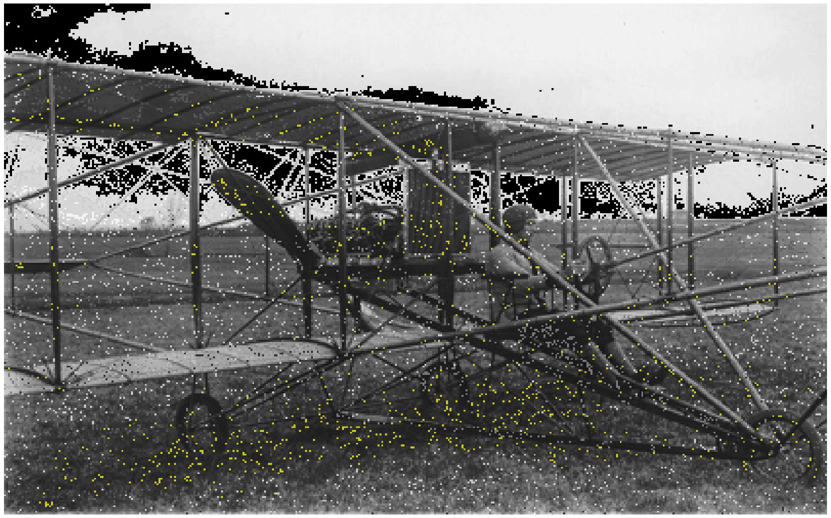
I really like the binomial method. But only as a teaching aid. It is the easiest way to explain

1. hedging to eliminate risk
2. no arbitrage
3. risk neutrality

I use it in the CQF to explain these important, and sometimes difficult to grasp, concepts.* But once the CQFers have understood these concepts they are instructed never to use the binomial model again, on pain of having their CQFs withdrawn!

*It's also instructive to also take a quick look at the trinomial version, because then you see immediately how difficult it is to hedge in practice.

The binomial model was the first of what are now known as finite-difference methods. It dates back to 1911 and was the creation of Lewis Fry Richardson, all-round mathematician, sociologist, and poet.



Things have come on a long way since 1911.

A lot of great work has been done on the development of these numerical methods in the last century. But very little has been done, relatively speaking, on the development of the binomial model. The binomial model is finite differences with one hand tied behind its back, hopping on one leg, while blindfolded.

- Lazy professors and their lecture notes

And so generations of students are led to believe that the binomial method is state of the art when it is actually prehistoric.

Summary

Question 1: What are the advantages of diversification among products, or even among mathematical models?

Answer 1: No advantage to your pay whatsoever!

Question 2: If you add risk and curvature what do you get?

Answer 2: Value!

Question 3: If you increase volatility what happens to the value of an option?

Answer 3: It depends on the option!

Question 4: If you use ten different volatility models to value an option and they all give you very similar values what can you say about volatility risk?

Answer 4: You may have a lot more than you think!

Question 5: One apple costs 50p, how much will 100 apples cost you?

Answer 5: Not £50!

QF is interesting and challenging, not because the mathematics is complicated, it isn't, but because putting maths and trading and market imperfections and human nature together and trying to model all this, knowing all the while that it is probably futile, now that's fun!

References

Ahmad, R & Wilmott, P 2005 Which free lunch would you like today, Sir? Delta hedging, volatility arbitrage and optimal portfolios. *Wilmott* magazine November 64–79

Ahmad, R & Wilmott, P 2007 The market price of interest rate risk: Measuring and modelling fear and greed in the fixed-income markets. *Wilmott* magazine, January

Ahn, H & Wilmott, P 2003 Stochastic volatility and mean-variance analysis. *Wilmott* magazine November 2003 84–90

Ahn, H & Wilmott, P 2007 Jump Diffusion, Mean and Variance: How to Dynamically Hedge, Statically Hedge and to Price. *Wilmott* magazine May 2007 96–109

Ahn, H & Wilmott, P 2008 Dynamic Hedging is Dead! Long Live Static Hedging! *Wilmott* magazine January 2008 80–7

Avellaneda, M, Levy, A & Parás, A 1995 Pricing and hedging derivative securities in markets with uncertain volatilities. Applied Mathematical Finance **2** 73–88

Avellaneda, M & Parás, A 1996 Managing the volatility risk of derivative securities: the Lagrangian volatility model. Applied Mathematical Finance **3** 21–53

Avellaneda, M & Buff, R 1997 Combinatorial implications of nonlinear uncertain volatility models: the case of barrier options. Courant Institute, NYU

Derman, E & Kani, I 1994 Riding on a smile. Risk magazine **7** (2) 32–39 (February)

Dupire, B 1993 Pricing and hedging with smiles. Proc AFFI Conf, La Baule June 1993

Dupire, B 1994 Pricing with a smile. Risk magazine **7** (1) 18–20 (January)

Haug, EG 2007 *Complete Guide to Option Pricing Formulas.* McGraw–Hill

Ho, T & Lee, S 1986 Term structure movements and pricing interest rate contingent claims. Journal of Finance **42** 1129–1142

Hoggard, T, Whalley, AE & Wilmott, P 1994 Hedging option portfolios in the presence of transaction costs. Advances in Futures and Options Research **7** 21–35

Hua, P & Wilmott, P 1997 Crash courses. Risk magazine **10** (6) 64–67 (June)

Hua, P & Wilmott, P 1998 Value at risk and market crashes. *Derivatives Week*

Hua, P & Wilmott, P 1999 Extreme scenarios, worst cases, CrashMetrics and Platinum Hedging. *Risk Professional*

Hua, P & Wilmott, P 2001 CrashMetrics. In *New Directions in Mathematical Finance*. (Eds P.Wilmott and H.Rasmussen.)

Hull, JC & White, A 1990 Pricing interest rate derivative securities. Review of Financial Studies **3** 573–592

Schönbucher, PJ & Wilmott, P 1995 Hedging in illiquid markets: nonlinear effects. Proceedings of the 8th European Conference on Mathematics in Industry

Schoutens, W, Simons, E & Tistaert, J 2004 A perfect calibration! Now what? *Wilmott* magazine March 2004 66–78

Rubinstein, M 1994 Implied binomial trees. Journal of Finance **69** 771–818

Vasicek, OA 1977 An equilibrium characterization of the term structure. Journal of Financial Economics **5** 177–188

Whalley, AE & Wilmott, P 1993 a Counting the costs. Risk magazine **6** (10) 59–66 (October)

Whalley, AE & Wilmott, P 1993 b Option pricing with transaction costs. MFG Working Paper, Oxford

Whalley, AE & Wilmott, P 1994 a Hedge with an edge. Risk magazine **7** (10) 82–85 (October)

Whalley, AE & Wilmott, P 1994 b A comparison of hedging strategies. Proceedings of the 7th European Conference on Mathematics in Industry 427–434

Whalley, AE & Wilmott, P 1995 An asymptotic analysis of the Davis, Panas and Zariphopoulou model for option pricing with transaction costs. MFG Working Paper, Oxford University

Whalley, AE & Wilmott, P 1996 Key results in discrete hedging and transaction costs. In *Frontiers in Derivatives* (Ed. Konishi, A and Dattatreya, R.) 183–196

Whalley, AE & Wilmott, P 1997 An asymptotic analysis of an optimal hedging model for option pricing with transaction costs. Mathematical Finance **7** 307–324

Wilmott, P 2002 Cliquet options and volatility models. *Wilmott* magazine December 2002

Wilmott, P 2006a *Paul Wilmott On Quantitative Finance, second edition*. John Wiley and Sons

Wilmott, P 2006b *Frequently Asked Questions in Quantitative Finance*. John Wiley and Sons