Expected Utility Asset Allocation

William F. Sharpe

STANCO 25 Professor of Finance
Stanford University
www.wsharpe.com

Reference

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Quantitative Approaches to Asset Allocation

- Mean/Variance
  - Markowitz quadratic programming optimizers
  - The Capital Asset Pricing Model of the relationships between risks and returns, used when estimating asset expected returns
- Expected Utility
  - A more general approach
    - Mean/variance is a special case
  - Used in academic analyses of asset pricing
  - Rarely used to make asset allocation decisions

Mean/Variance Asset Allocation

- Focuses only on portfolio:
  - Expected Return (mean)
  - Risk (Standard deviation or variance)

- Rationales:
  - Investor preferences
    - Investors care only about portfolio mean and variance
  - Portfolio Returns
    - Mean and variance are sufficient statistics
    - Knowing them one can determine the entire distribution of returns
    - For example, all distributions might be normal (bell-shaped)

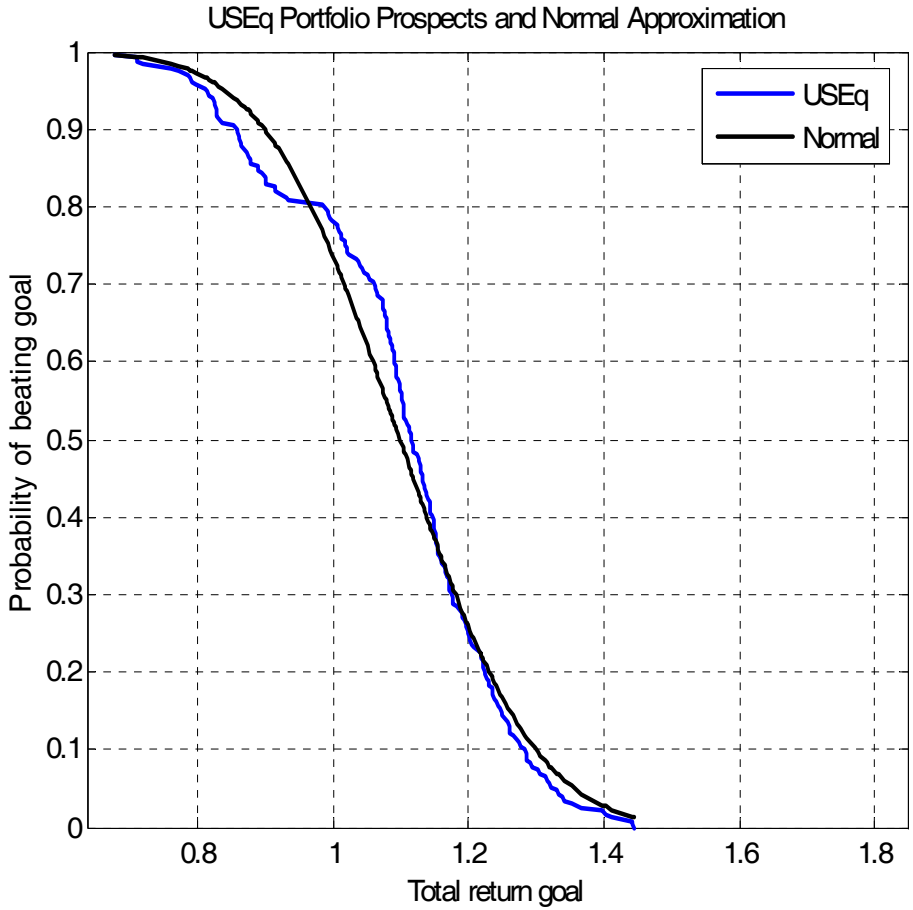
Investor Preferences

- Investors care about tail risk, extreme events, etc.
- Unless mean and variance are sufficient statistics, they may not provide enough information for choosing an asset allocation

Portfolio Returns

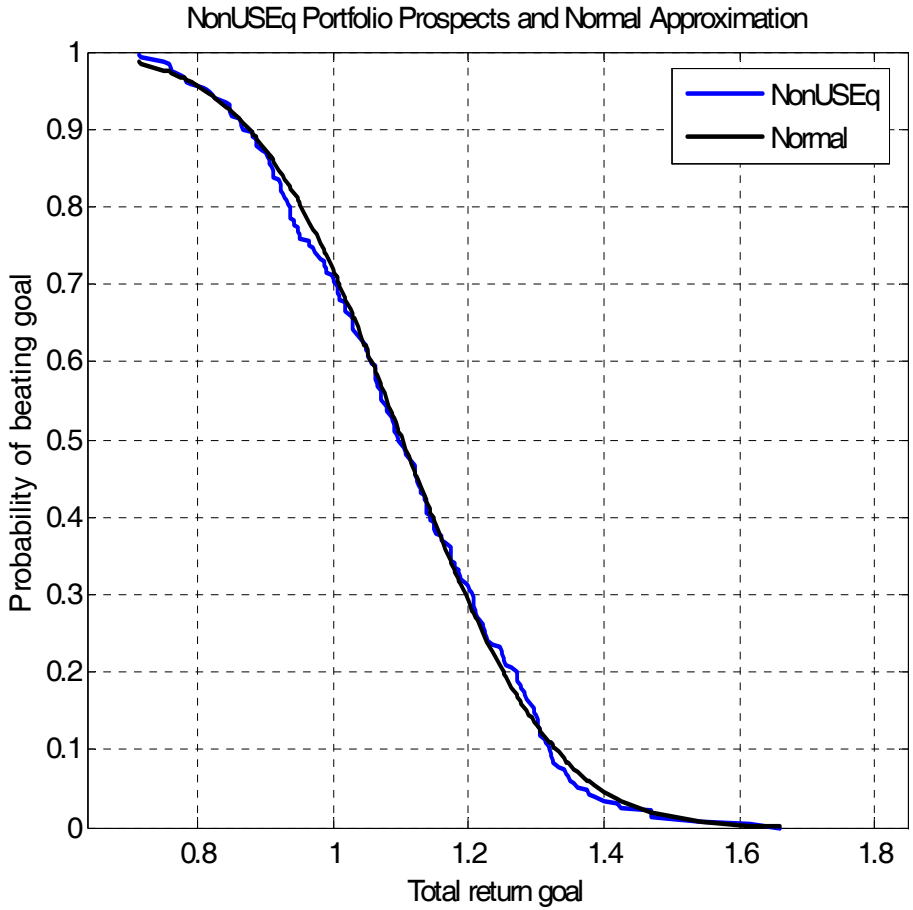
- Some asset classes and portfolios are approximately normally distributed
  - they can be described relatively well by mean and standard deviation or variance
- Other asset classes and portfolios have substantially non-normal distributions
  - mean and standard deviation or variance may not suffice for making decisions

U.S. Equity Portfolio Returns



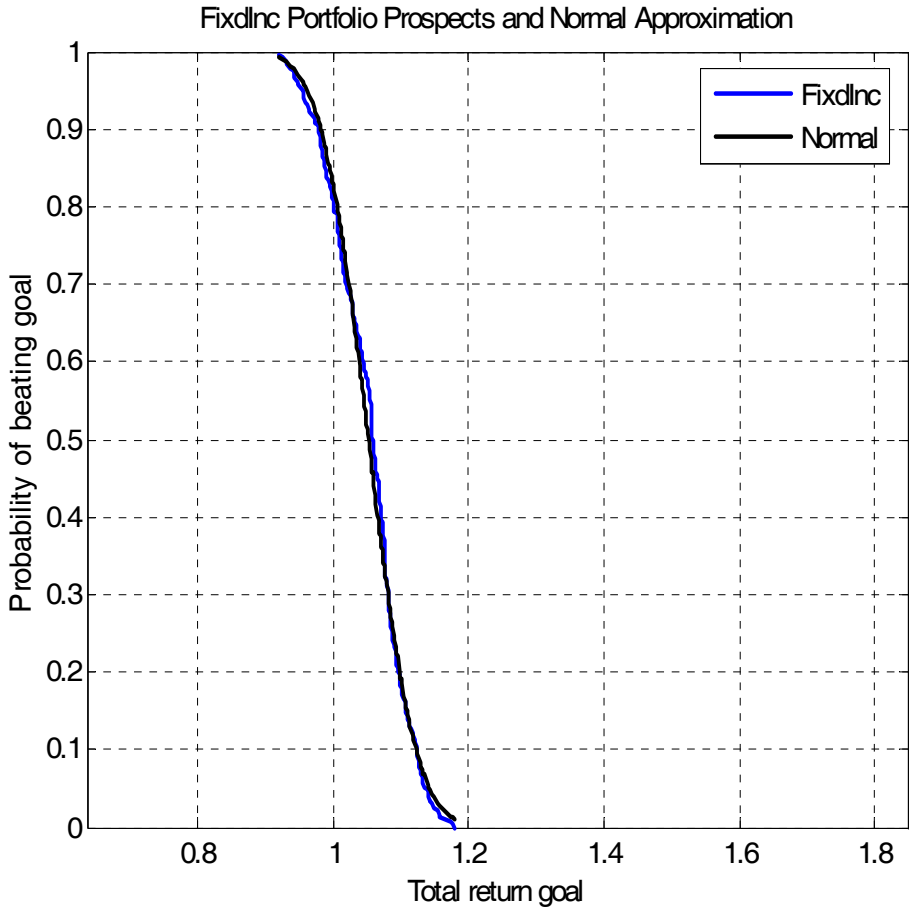
Wilshire 5000
1987-2006 overlapping years with EU equilibrium adjustment

Non-US Equity Portfolio Returns



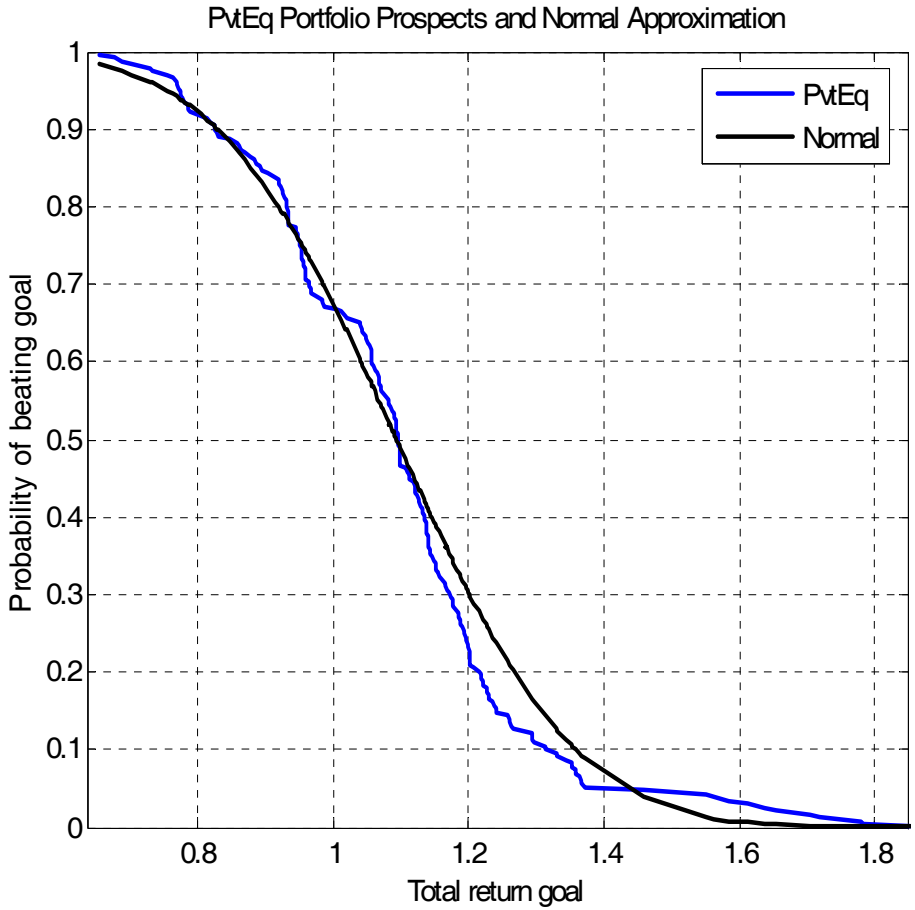
FTSE and MSCI All-World ex US
1987-2006 overlapping years with EU equilibrium adjustment

Global Fixed Income Portfolio Returns



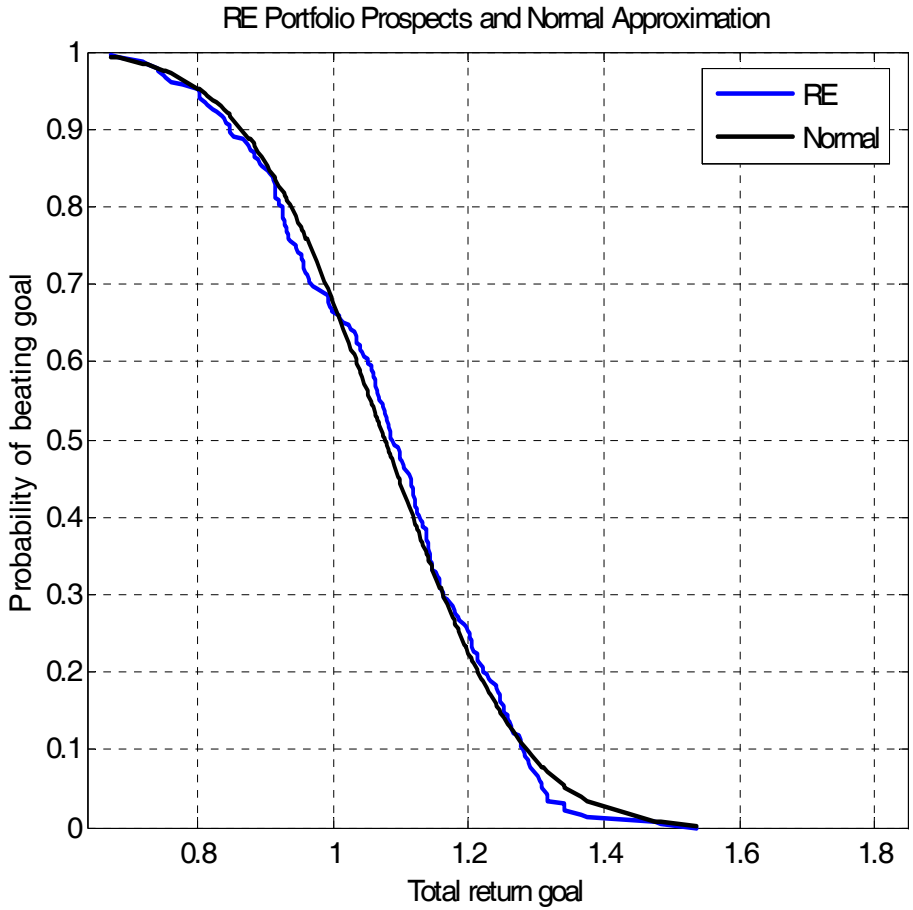
Salomon Brothers' Indices
1987-2006 overlapping years with EU equilibrium adjustment

Private Equity Portfolio Returns



Large Pension Fund Custom Benchmark
1987-2006 overlapping years with EU equilibrium adjustment

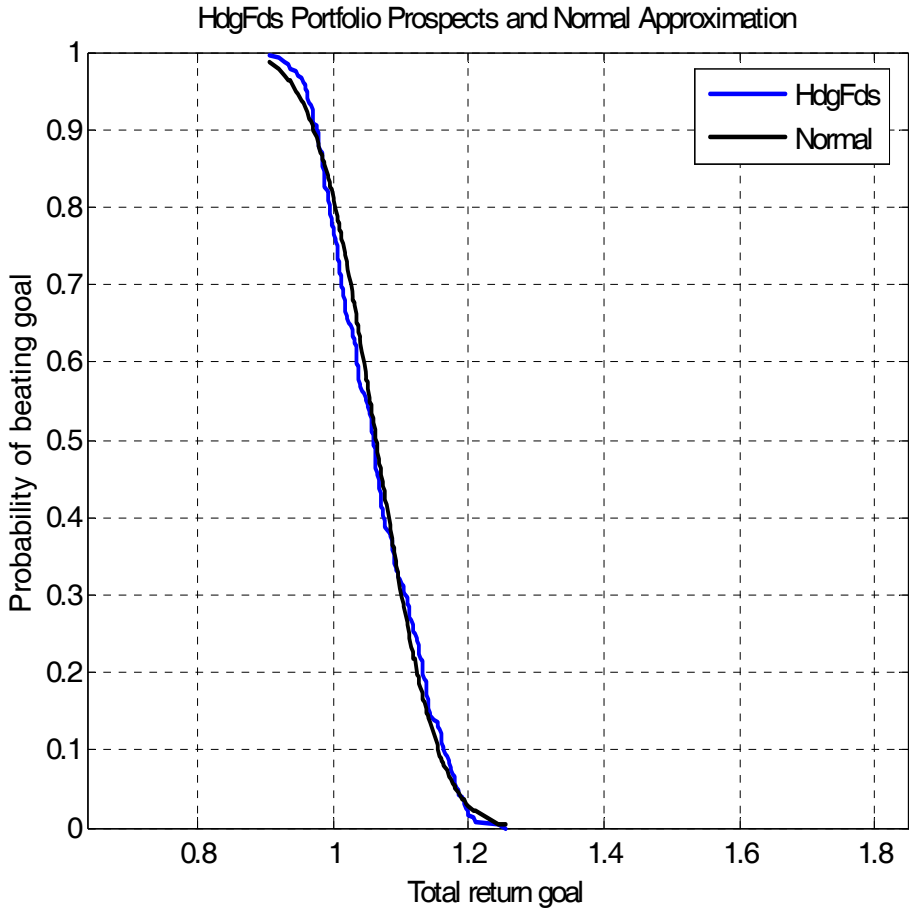
Real Estate Portfolio Returns



US REITs

1987-2006 overlapping years with EU equilibrium adjustment

Hedge Fund Index Portfolio Returns



HFN Aggregate Average Index
1987-2006 overlapping years with EU equilibrium adjustment

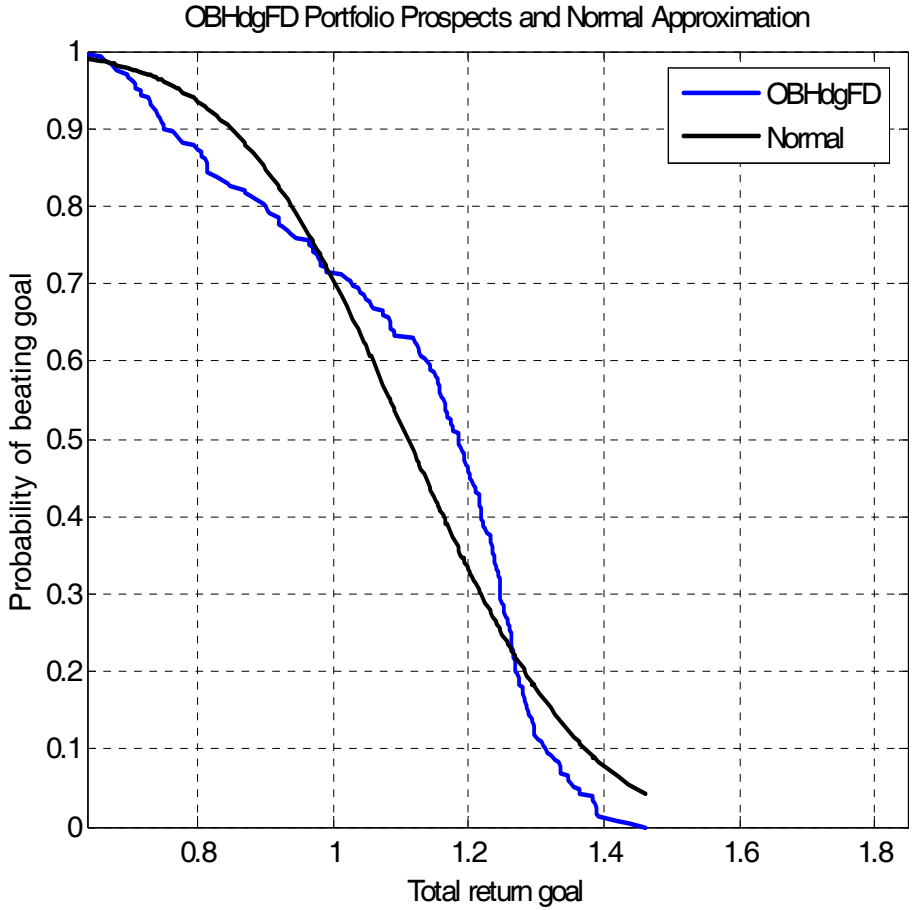
Hedge Fund Returns

- May have small probability of very negative returns
  - “picking up nickels in front of a steamroller
  - non-normal returns with substantial tail risk
- The majority of indices of hedge fund returns are biased
  - Only surviving funds are included
  - Funds with poor records, even if still in business, are less likely to have provided data for the index
  - Equal-weighted or median returns are not likely to be representative of the return on the average dollar (euro, yen …) invested in such funds

Replicating the Returns of Some Hedged Strategies

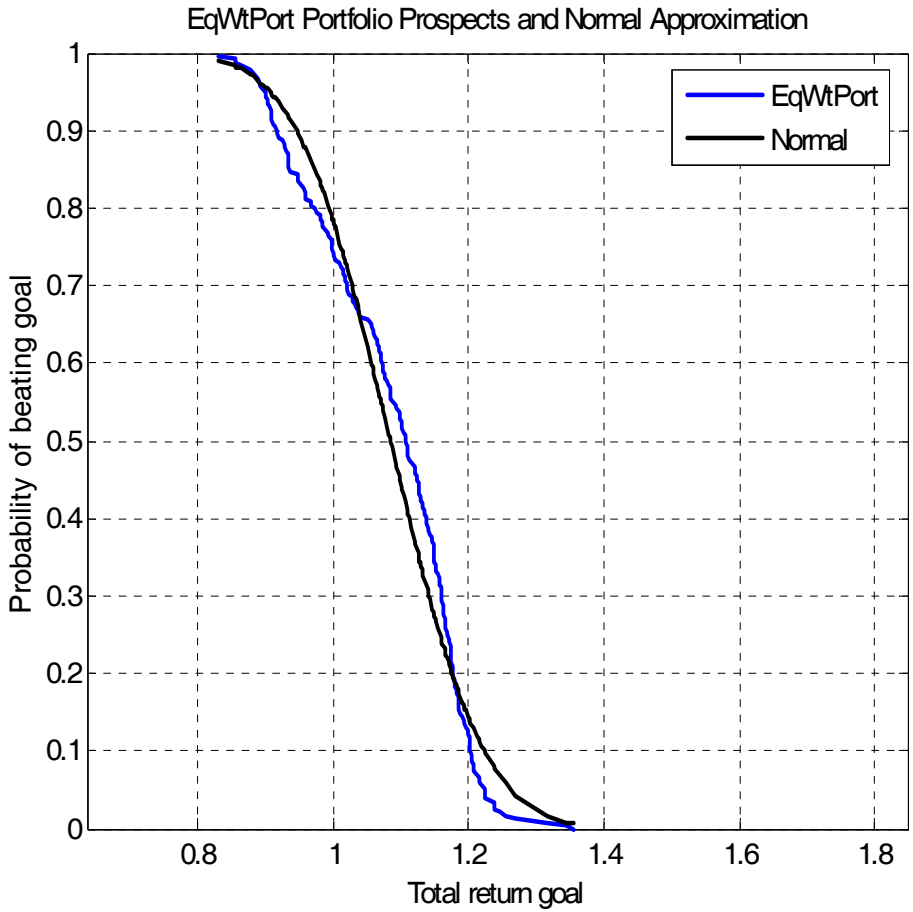
- Sell out-of-the-money puts on a stock market
- Invest initial funds plus proceeds from the sale of the puts in in marketed indices
  - For example, 1/3 in stocks and 2/3 in cash equivalents
- In all periods but those with very bad stock market returns, such a fund will have superior performance
  - in particular, a high Sharpe Ratio
- The overall distribution of returns for such a strategy may be similar to that of some hedged strategies
  - *Capital Decimation Partners* in Andrew Lo, "Risk Management for Hedge Funds: Introduction and Overview," *Financial Analysts Journal*, 2001

Option-based Hedge Fund Portfolio Returns



33% US equity with 2X 7% OOM Put Option
1987-2006 overlapping years with EU equilibrium adjustment

Diversified Portfolio Returns



Portfolio with Equal Weights in each of 7 asset classes
1987-2006 overlapping years with EU equilibrium adjustment

Paul Samuelson's Opinion

- "Markowitz-Sharpe-Tobin quadratic programming in terms of portfolio means and variances is a powerful approximation that has captured real-world converts the way that smallpox used to infect once-isolated aborigines."

  – Paul A. Samuelson, "The Backward Art of Investing Money," *Journal of Portfolio Management*, Sept. 2004.

Alternative Approaches to Standard Mean/Variance Asset Allocation

- Constrained mean/variance analysis
  - Upper and lower bounds on some or all assets
- Augment with additional analyses to measure tail risk, etc.
  - stress tests
- Add further statistics to mean and variance
  - Skewness, etc.
- Return to fundamentals
  - Expected Utility Asset Allocation

Expected Utility Asset Allocation

- The starting point for Mean/Variance analysis
- Allows for more realism
  - Different types of return distributions
  - Different types of investor preferences
- Not a “paradigm shift”
- A natural progression as theory and practice expand to take more aspects of reality into account

Expected Utility Analysis: Advantages

- Can take into account attitudes about extreme returns, departures from target return, etc.
- Uses a single, integrated approach rather than a series of separate analyses
- Can accommodate views about different probabilities of scenarios and can incorporate scenarios that did not occur in the past

Expected Utility Analysis: Disadvantages

- Requires scenarios representing a sufficiently wide range of asset returns
- Requires explicit representation of attitudes about different levels of return
- More degrees of freedom
  - Can get more better results
  - Could get worse results

Asset Allocation Procedures

- Optimization
  - Prescriptive
  - What asset allocation is best for a specific investor?
- Reverse Optimization
  - Descriptive
  - What are the opportunities in the capital markets?

Optimization

- Given
  - Plausible estimates of capital market opportunities
  - The preferences of a specific investor
- Find:
  - The optimal asset allocation for that investor

Reverse Optimization

- Given:
  - Historic asset returns
  - Current asset market values
  - Assumptions about the average preferences of all investors
- Find:
  - Plausible estimates of capital market opportunities

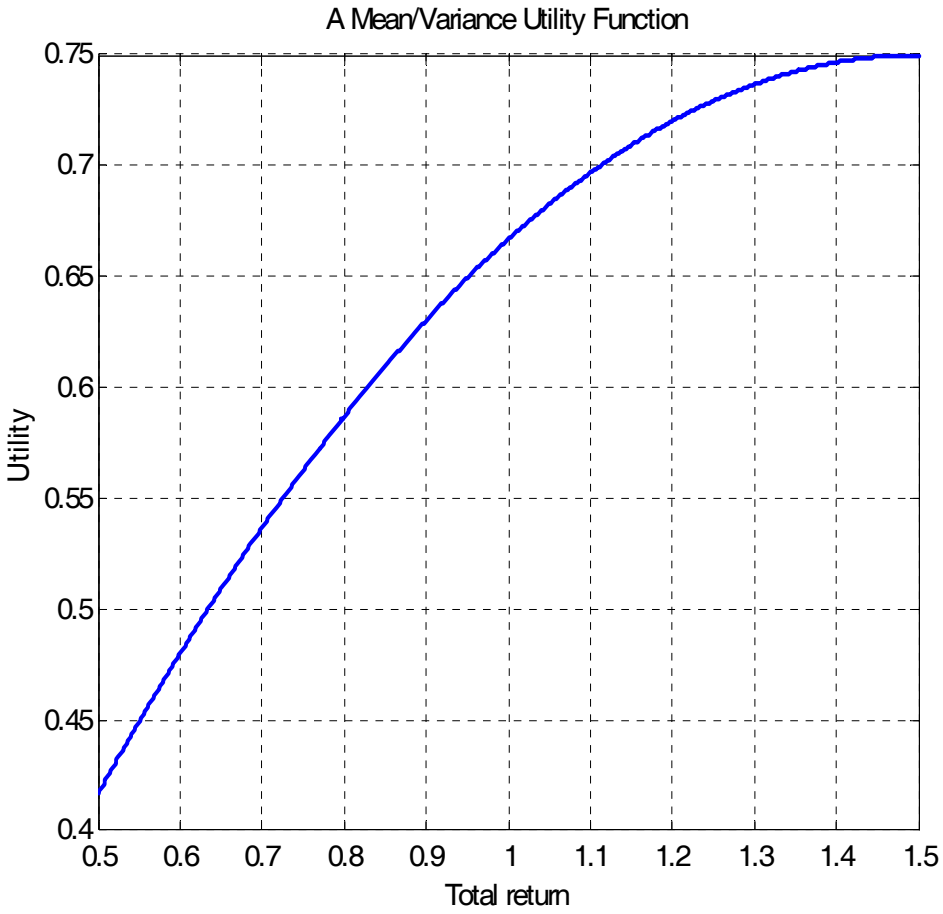
Expected Utility Optimization

- Goal:
  - Find the asset allocation that provides the maximum possible expected utility (EU) for an investor
- Utility
  - A measure of the happiness a particular portfolio return would provide the investor in question
- Expected Utility
  - A weighted average of the utilities of all possible portfolio returns using the probabilities of the returns as weights

Maximizing Expected Utility

- Start with a feasible allocation
- Find the best buy
  - Maximum increase in EU per $ bought
- Find the best sell
  - Minimum decrease in EU per $ sold
- Sell $x of the best sell, buy $x of the best buy
  - Select $x to maximize the net gain in EU
- Continue until no further improvement is possible

A Mean/Variance Investor's Utility Function

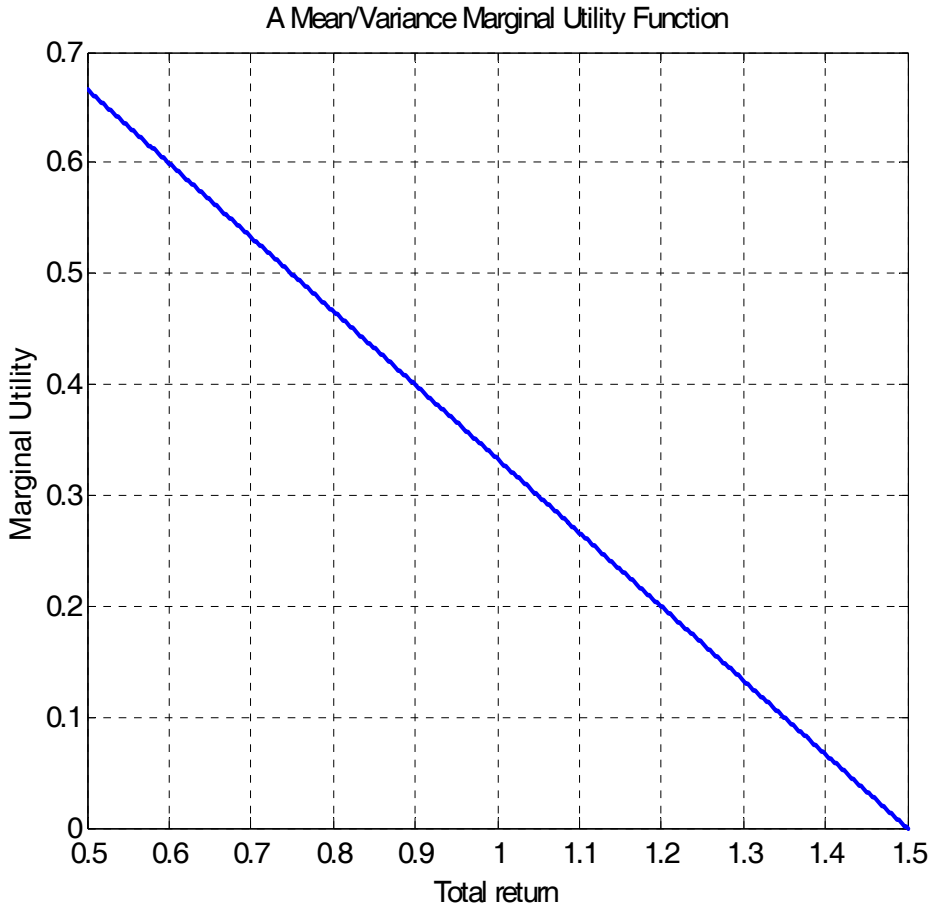


Quadratic Utility with Satiation = 1.50

Marginal Utility

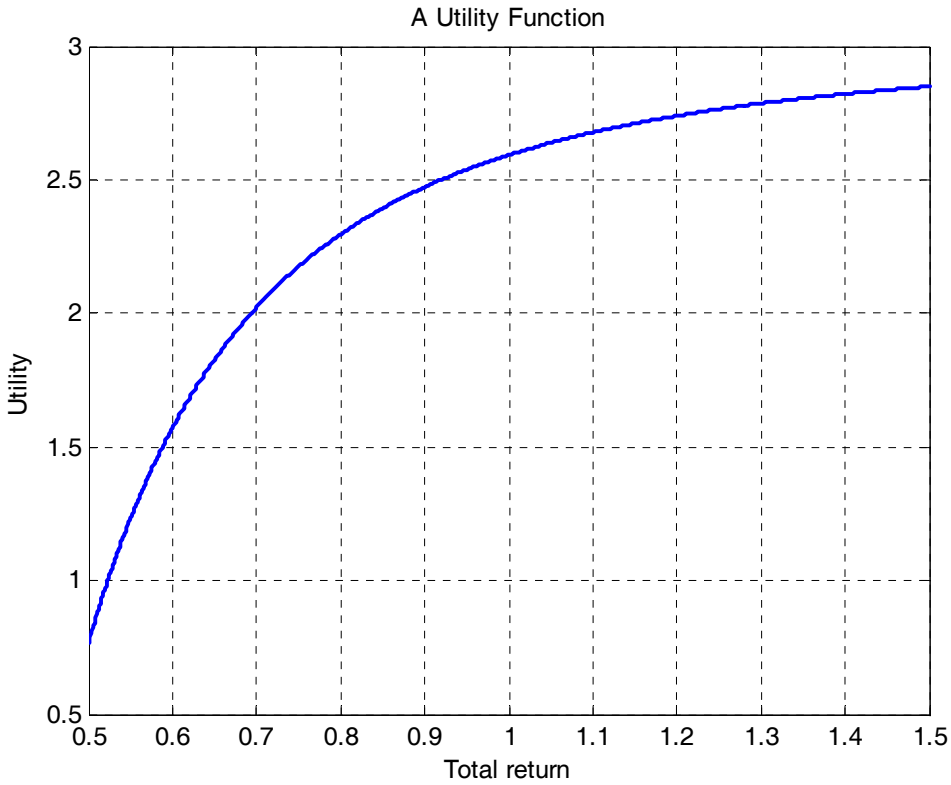
- The added (loss of) utility from a small increase (decrease) in return
- For risk-averse investors:
  - When return is low, marginal utility is high
  - When return is high, marginal utility is low

A Mean/Variance Investor's Marginal Utility Function



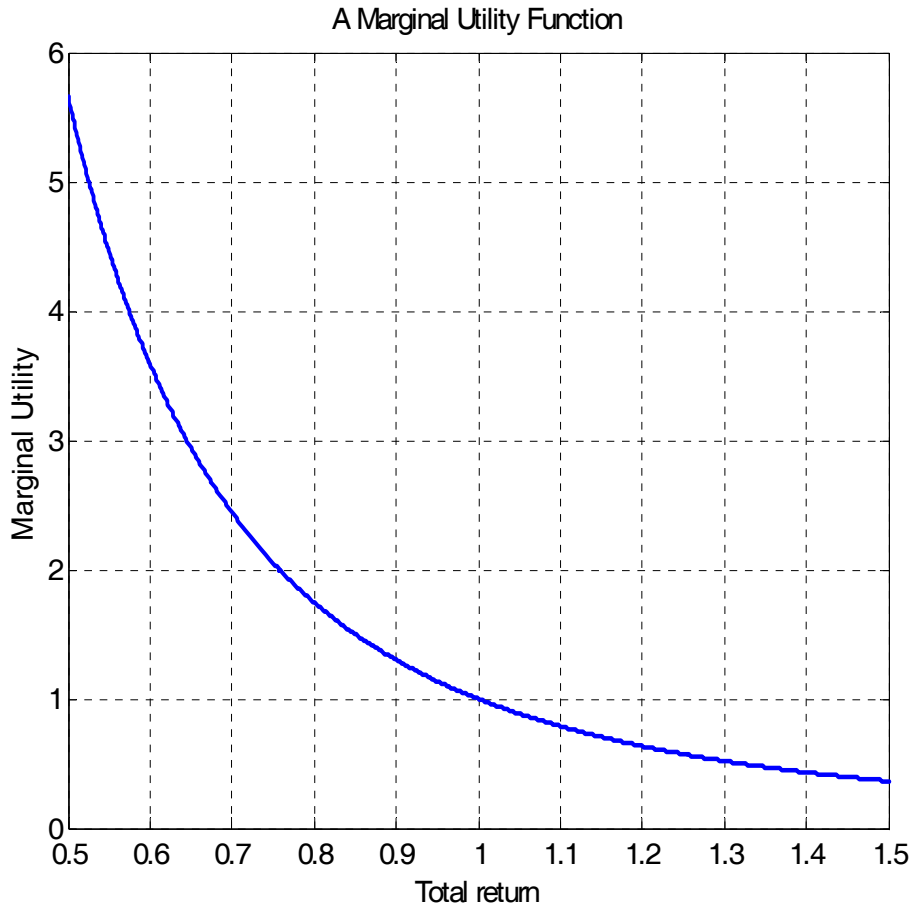
Quadratic Utility with Satiation = 1.50

A More Typical Investor's Utility Function



Constant Relative Risk-aversion with ra = 3.5

A More Typical Investor's Marginal Utility Function



Constant Relative Risk-aversion with ra = 3.5

Specifying an Investor's Preferences

- For many investors a mean/variance utility function may represent a good approximation of true preferences
  - If so, expected utility maximization will provide results only slightly better than those from a standard Markowitz optimization
- But for at least some investors a mean/variance utility function may be a poor approximation of , true preferences
  - And expected utility maximization could provide significantly better results

The Need for Reverse Optimization

- Asset allocation decisions should be based on predicted future returns
- Historic returns can be useful for predicting future uncertainty and interrelationships among asset returns
- However, historic average returns are not likely to be the best predictors of expected future returns
- To deal with this, analysts often adjust historic returns so the resulting predictions will be wholly or partly consistent with assumptions about capital market efficiency

The Market Portfolio

- Includes each asset in an amount proportional to its current market value
- Reflects current forecasts of future asset returns
- Provides valuable information that should be utilized when making asset return forecasts
- A number of economic models conclude that in an efficient capital market the market portfolio will be optimal for an investor with “representative preferences”
  - For more, see William F. Sharpe, *Investors and Markets: Portfolio Choices, Asset Prices and Investment Advice*, Princeton University Press, 2007

Reverse Optimization

- For each asset, adjust historic asset returns by adding (subtracting) a constant to (from) every historic return so the market portfolio will be optimal for an investor with representative preferences

- If desired, the constants can be modified to reflect an analyst's views about asset mispricing

Specifying the Representative Investor's Preferences

- If the representative investor's preferences are approximated with a mean/variance utility function, expected utility reverse optimization will give the same results as the Capital Asset Pricing Model

- But, if a different approximation of the representative investor's preferences is utilized, expected utility maximization may provide a more realistic set of possible future asset returns
  - This can lead, in turn, to more realistic predictions of likely future portfolio returns and better asset allocations

Now and Then

- There are now many more investment vehicles with complex return distributions:
  - Alternative investments
  - Hedge funds
  - Derivatives

- We now know more about the preferences of:
  - Individual investors and institutions, and
  - A representative investor reflecting the preferences of all investors and institutions

Conclusions

- There is no need to make the restrictive assumptions associated with mean/variance analyses

- Thus for cases in which investor preferences and/or return characteristics make the mean/variance approach inferior, the expected utility approach should be seriously considered as an alternative