

# Expected Utility Asset Allocation

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# Reference

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# Quantitative Approaches to Asset Allocation

- Mean/Variance
  - Markowitz quadratic programming optimizers
  - The Capital Asset Pricing Model of the relationships between risks and returns, used when estimating asset expected returns
- Expected Utility
  - A more general approach
    - Mean/variance is a special case
  - Used in academic analyses of asset pricing
  - Rarely used to make asset allocation decisions

# Mean/Variance Asset Allocation

- Focuses only on portfolio:
  - Expected Return (mean)
  - Risk (Standard deviation or variance)
- Rationales:
  - Investor preferences
    - Investors care only about portfolio mean and variance
  - Portfolio Returns
    - Mean and variance are sufficient statistics
    - Knowing them one can determine the entire distribution of returns
    - For example, all distributions might be normal (bell-shaped)

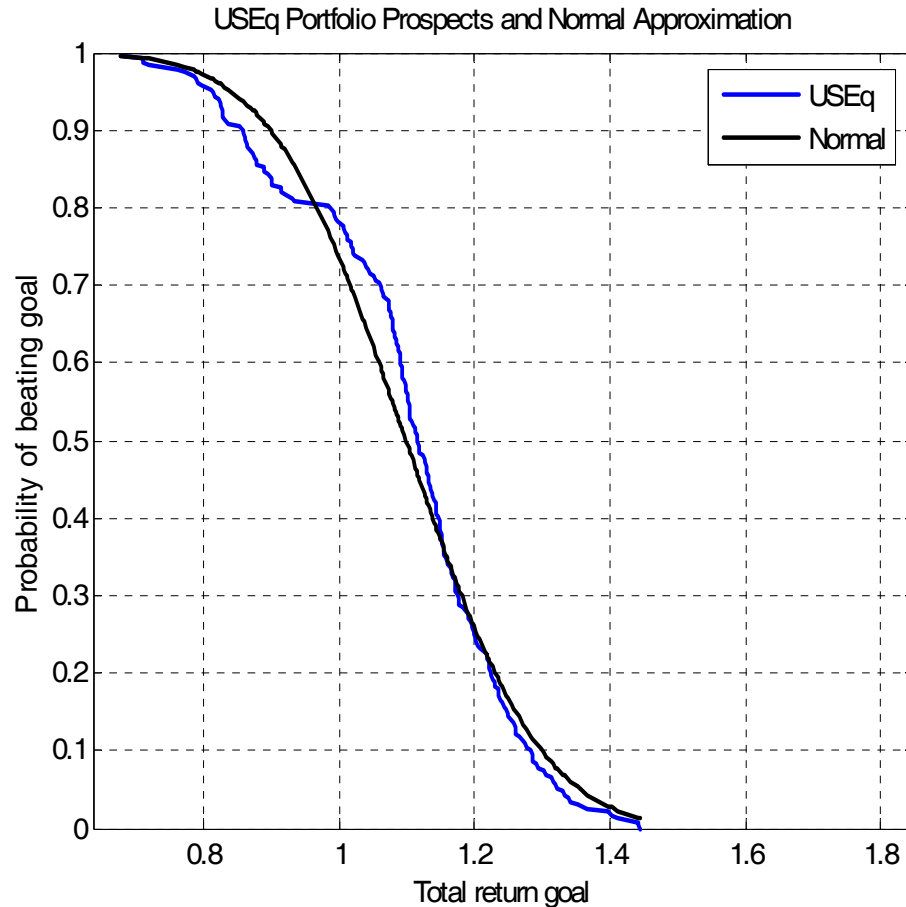
# Investor Preferences

- Investors care about tail risk, extreme events, etc.
- Unless mean and variance are sufficient statistics, they may not provide enough information for choosing an asset allocation

# Portfolio Returns

- Some asset classes and portfolios are approximately normally distributed
  - they can be described relatively well by mean and standard deviation or variance
- Other asset classes and portfolios have substantially non-normal distributions
  - mean and standard deviation or variance may not suffice for making decisions

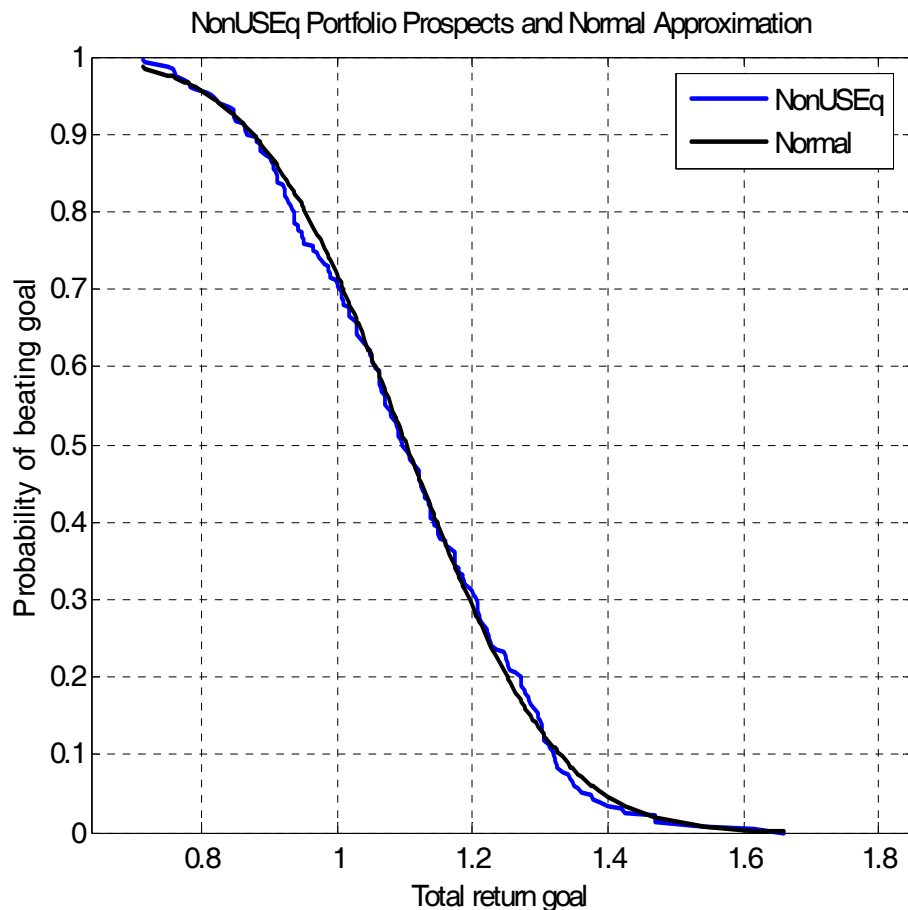
# U.S. Equity Portfolio Returns



Wilshire 5000

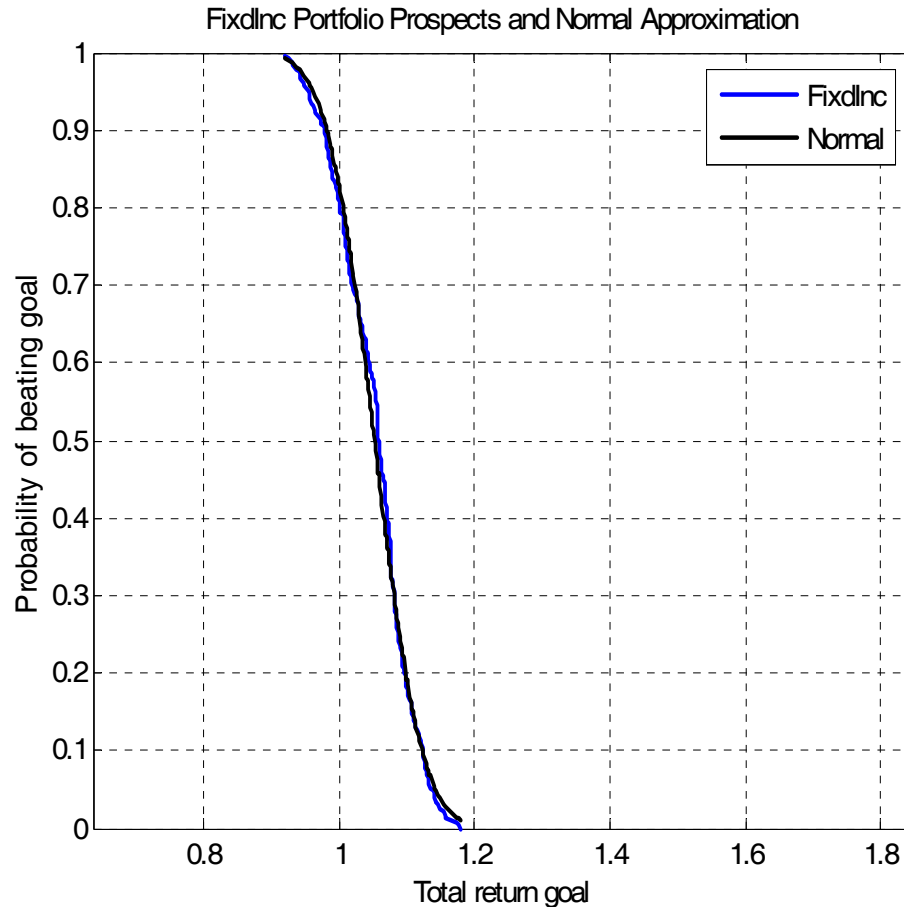
1987-2006 overlapping years with EU equilibrium adjustment

# Non-US Equity Portfolio Returns



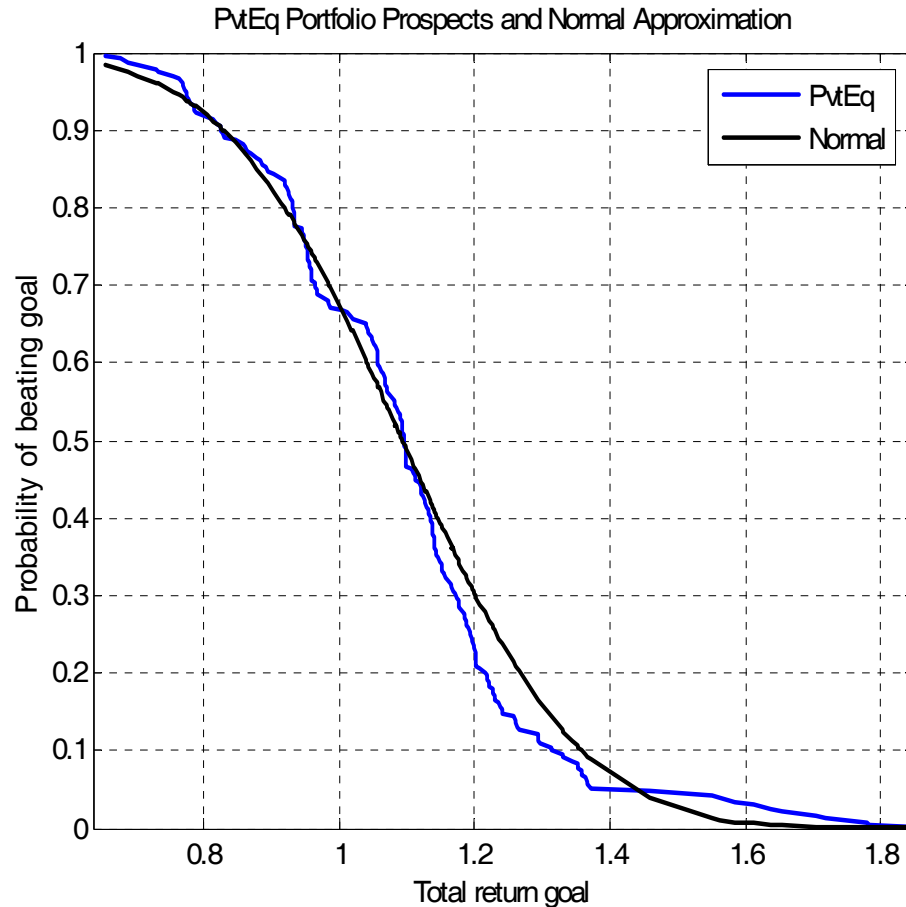
FTSE and MSCI All-World ex US  
1987-2006 overlapping years with EU equilibrium adjustment

# Global Fixed Income Portfolio Returns



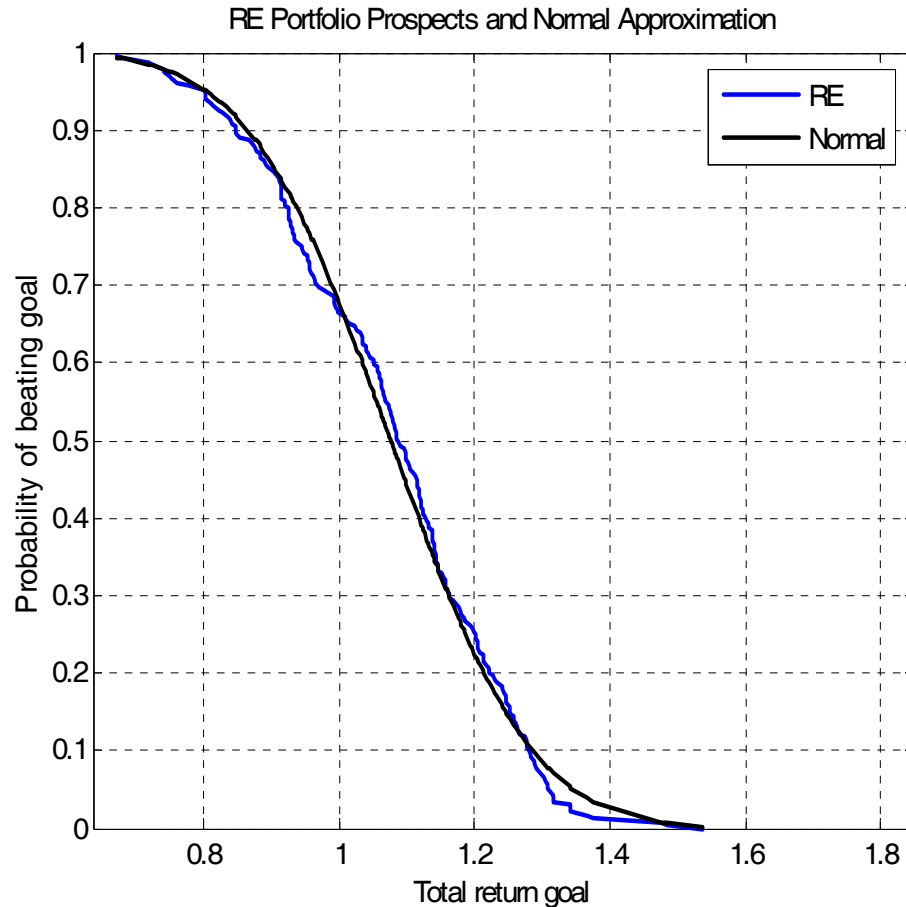
Salomon Brothers' Indices  
1987-2006 overlapping years with EU equilibrium adjustment

# Private Equity Portfolio Returns



Large Pension Fund Custom Benchmark  
1987-2006 overlapping years with EU equilibrium adjustment

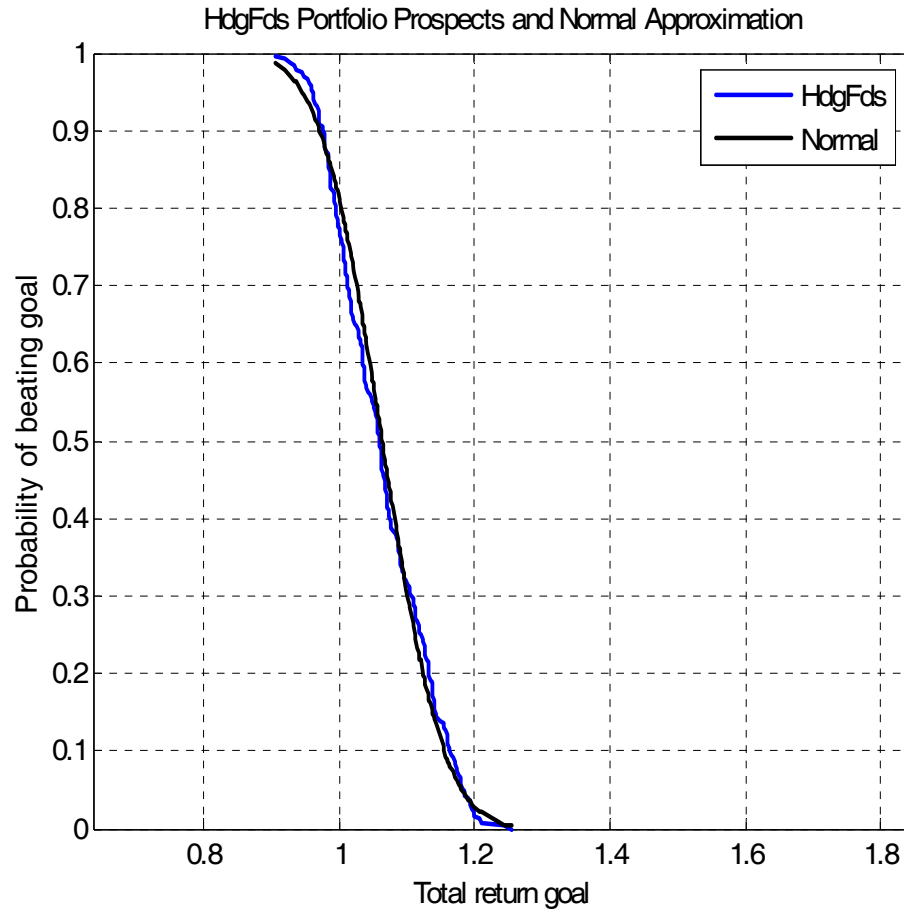
# Real Estate Portfolio Returns



US REITs

1987-2006 overlapping years with EU equilibrium adjustment

# Hedge Fund Index Portfolio Returns



HFN Aggregate Average Index  
1987-2006 overlapping years with EU equilibrium adjustment

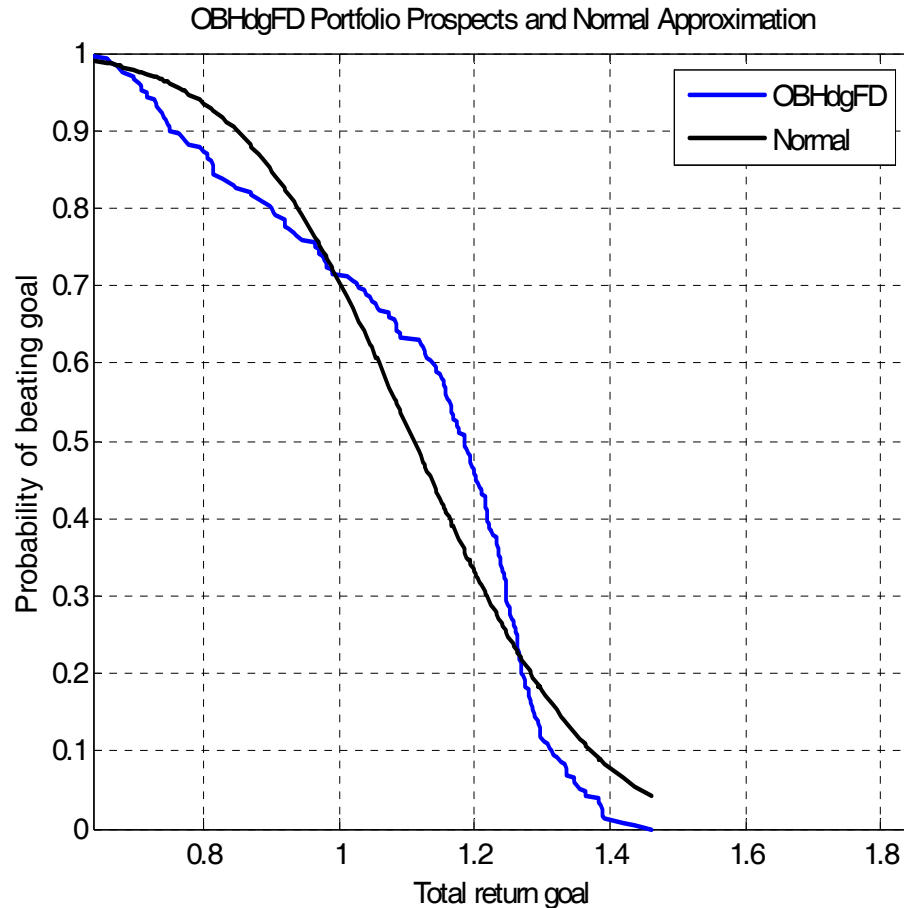
# Hedge Fund Returns

- May have small probability of very negative returns
  - “picking up nickels in front of a steamroller
  - non-normal returns with substantial tail risk
- The majority of indices of hedge fund returns are biased
  - Only surviving funds are included
  - Funds with poor records, even if still in business, are less likely to have provided data for the index
  - Equal-weighted or median returns are not likely to be representative of the return on the average dollar (euro, yen ...) invested in such funds

# Replicating the Returns of Some Hedged Strategies

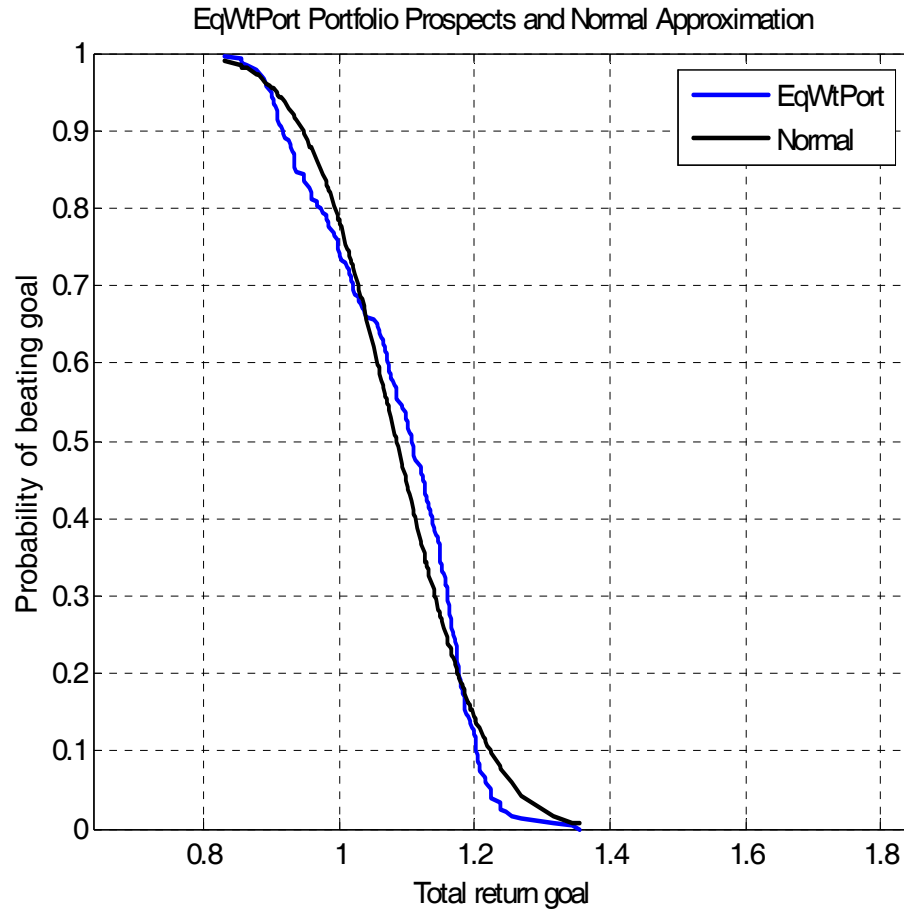
- Sell out-of-the-money puts on a stock market
- Invest initial funds plus proceeds from the sale of the puts in in marketed indices
  - For example, 1/3 in stocks and 2/3 in cash equivalents
- In all periods but those with very bad stock market returns, such a fund will have superior performance
  - in particular, a high Sharpe Ratio
- The overall distribution of returns for such a strategy may be similar to that of some hedged strategies
  - *Capital Decimation Partners* in Andrew Lo, “Risk Management for Hedge Funds: Introduction and Overview,” *Financial Analysts Journal*, 2001

# Option-based Hedge Fund Portfolio Returns



33% US equity with 2X 7% OOM Put Option  
1987-2006 overlapping years with EU equilibrium adjustment

# Diversified Portfolio Returns



Portfolio with Equal Weights in each of 7 asset classes  
1987-2006 overlapping years with EU equilibrium adjustment

# Paul Samuelson's Opinion

- “Markowitz-Sharpe-Tobin quadratic programming in terms of portfolio means and variances is a powerful approximation that has captured real-world converts the way that smallpox used to infect once-isolated aborigines.”
  - Paul A. Samuelson, “The Backward Art of Investing Money,” *Journal of Portfolio Management*, Sept. 2004.

# Alternative Approaches to Standard Mean/Variance Asset Allocation

- Constrained mean/variance analysis
  - Upper and lower bounds on some or all assets
- Augment with additional analyses to measure tail risk, etc.
  - stress tests
- Add further statistics to mean and variance
  - Skewness, etc.
- Return to fundamentals
  - Expected Utility Asset Allocation

# Expected Utility Asset Allocation

- The starting point for Mean/Variance analysis
- Allows for more realism
  - Different types of return distributions
  - Different types of investor preferences
- Not a “paradigm shift”
- A natural progression as theory and practice expand to take more aspects of reality into account

# Expected Utility Analysis: Advantages

- Can take into account attitudes about extreme returns, departures from target return, etc.
- Uses a single, integrated approach rather than a series of separate analyses
- Can accommodate views about different probabilities of scenarios and can incorporate scenarios that did not occur in the past

# Expected Utility Analysis: Disadvantages

- Requires scenarios representing a sufficiently wide range of asset returns
- Requires explicit representation of attitudes about different levels of return
- More degrees of freedom
  - Can get more better results
  - Could get worse results

# Asset Allocation Procedures

- Optimization
  - Prescriptive
  - What asset allocation is best for a specific investor?
- Reverse Optimization
  - Descriptive
  - What are the opportunities in the capital markets?

# Optimization

- Given
  - Plausible estimates of capital market opportunities
  - The preferences of a specific investor
- Find:
  - The optimal asset allocation for that investor

# Reverse Optimization

- Given:
  - Historic asset returns
  - Current asset market values
  - Assumptions about the average preferences of all investors
- Find:
  - Plausible estimates of capital market opportunities

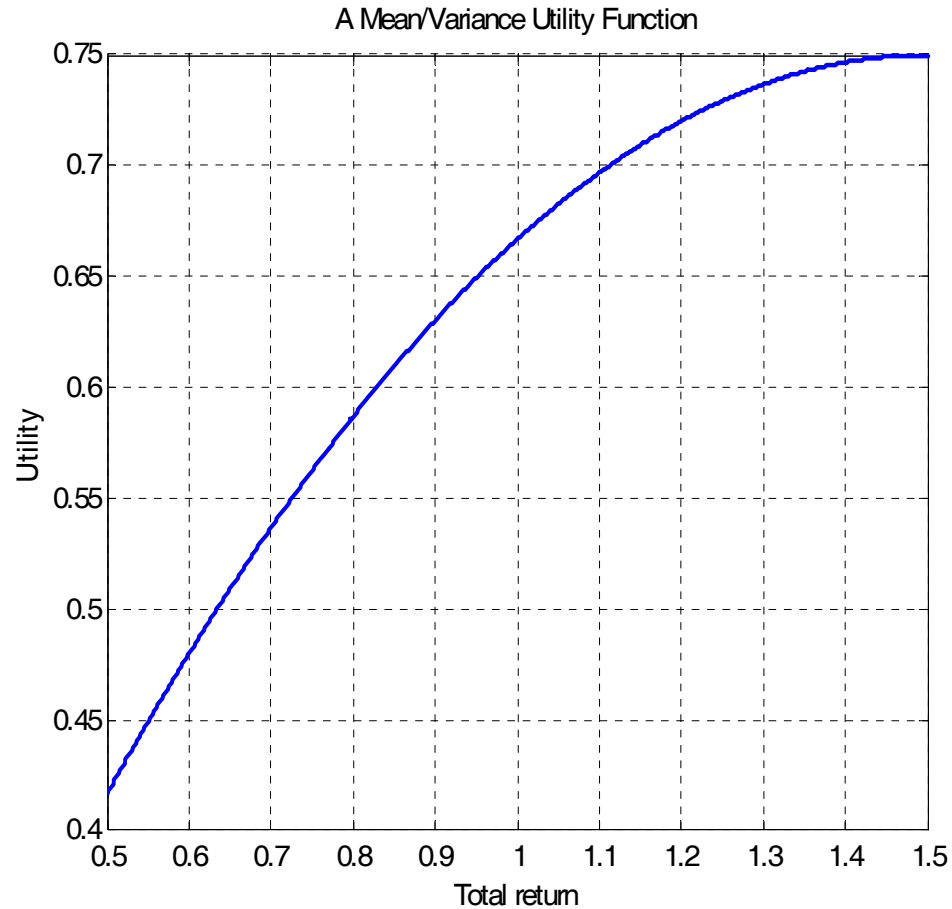
# Expected Utility Optimization

- Goal:
  - Find the asset allocation that provides the maximum possible expected utility (EU) for an investor
- Utility
  - A measure of the happiness a particular portfolio return would provide the investor in question
- Expected Utility
  - A weighted average of the utilities of all possible portfolio returns using the probabilities of the returns as weights

# Maximizing Expected Utility

- Start with a feasible allocation
- Find the best buy
  - Maximum increase in EU per \$ bought
- Find the best sell
  - Minimum decrease in EU per \$ sold
- Sell \$x of the best sell, buy \$x of the best buy
  - Select \$x to maximize the net gain in EU
- Continue until no further improvement is possible

# A Mean/Variance Investor's Utility Function

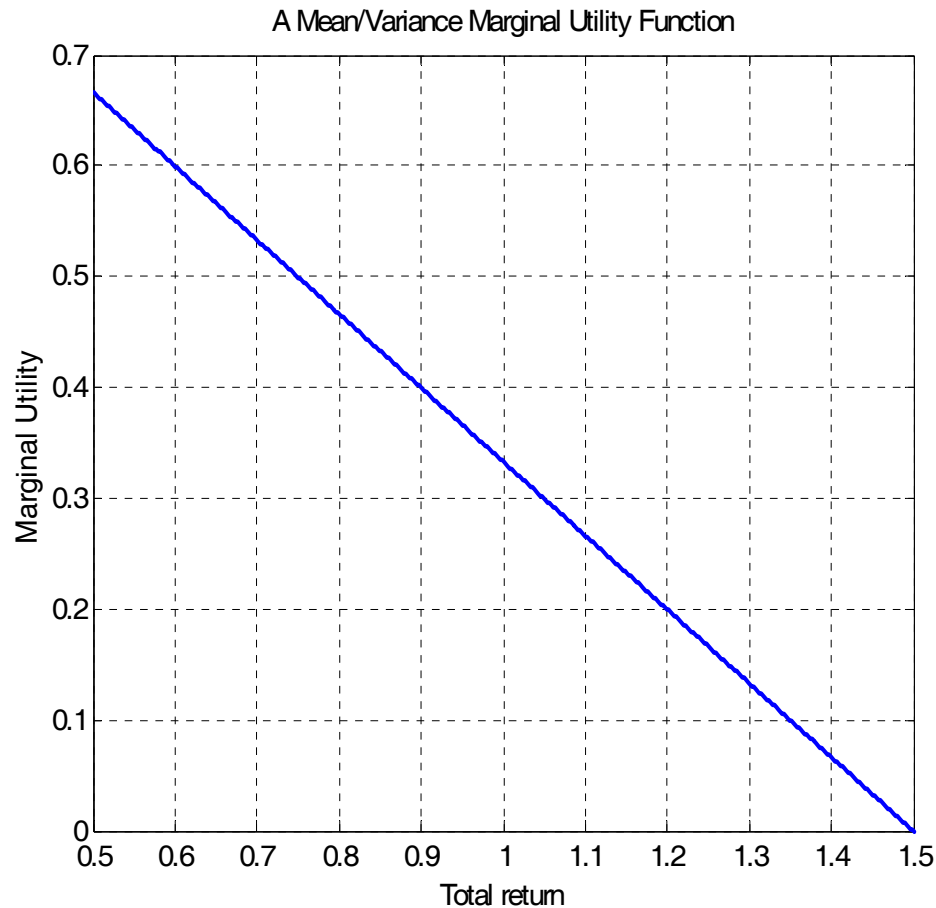


Quadratic Utility with Satiation = 1.50

# Marginal Utility

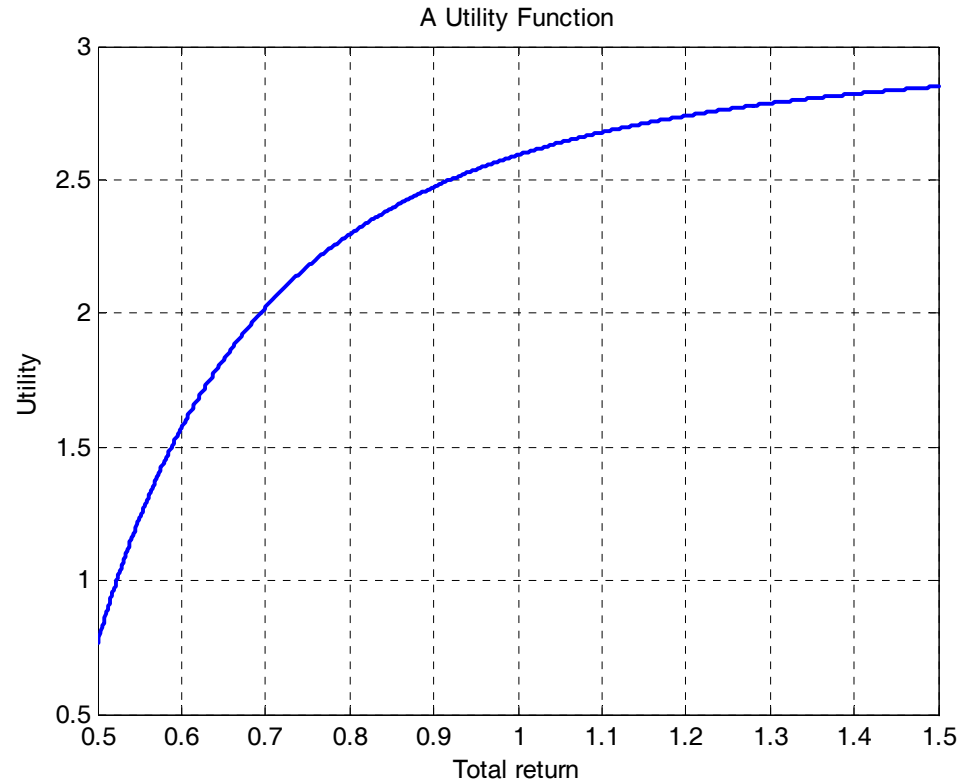
- The added (loss of) utility from a small increase (decrease) in return
- For risk-averse investors:
  - When return is low, marginal utility is high
  - When return is high, marginal utility is low

# A Mean/Variance Investor's Marginal Utility Function



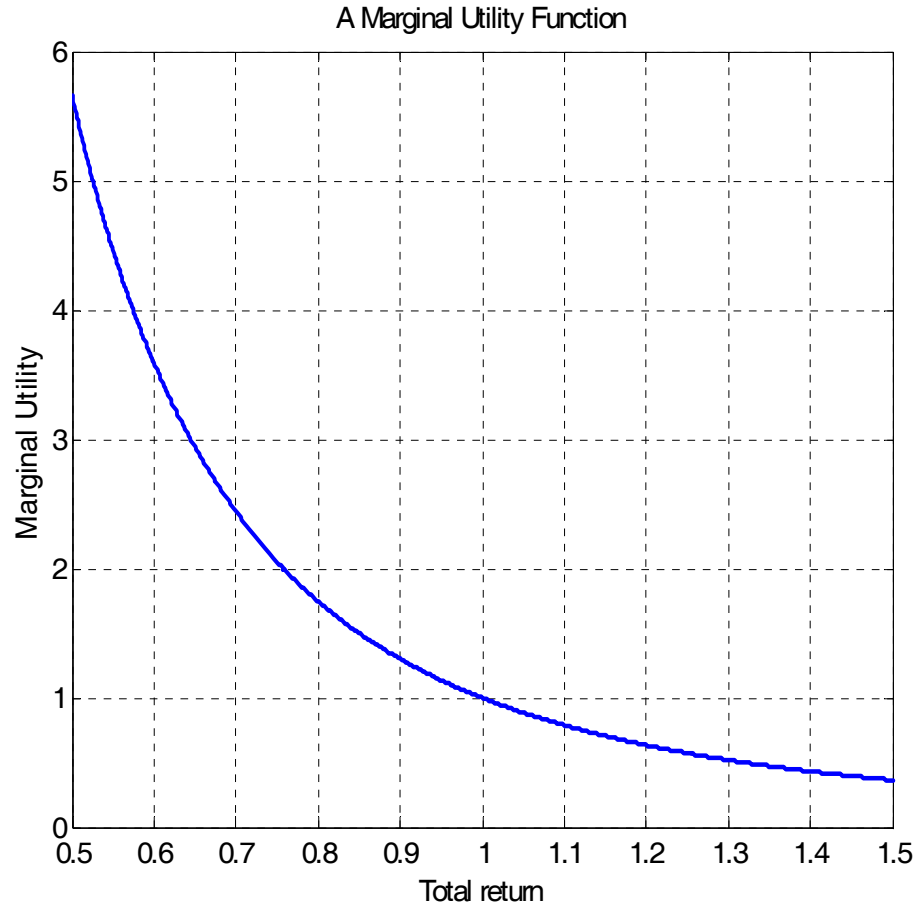
Quadratic Utility with Satiation = 1.50

# A More Typical Investor's Utility Function



Constant Relative Risk-aversion with  $ra = 3.5$

# A More Typical Investor's Marginal Utility Function



Constant Relative Risk-aversion with  $ra = 3.5$

# Specifying an Investor's Preferences

- For many investors a mean/variance utility function may represent a good approximation of true preferences
  - If so, expected utility maximization will provide results only slightly better than those from a standard Markowitz optimization
- But for at least some investors a mean/variance utility function may be a poor approximation of , true preferences
  - And expected utility maximization could provide significantly better results

# The Need for Reverse Optimization

- Asset allocation decisions should be based on predicted future returns
- Historic returns can be useful for predicting future uncertainty and interrelationships among asset returns
- However, historic average returns are not likely to be the best predictors of expected future returns
- To deal with this, analysts often adjust historic returns so the resulting predictions will be wholly or partly consistent with assumptions about capital market efficiency

# The Market Portfolio

- Includes each asset in an amount proportional to its current market value
- Reflects current forecasts of future asset returns
- Provides valuable information that should be utilized when making asset return forecasts
- A number of economic models conclude that in an efficient capital market the market portfolio will be optimal for an investor with “representative preferences”
  - For more, see William F. Sharpe, *Investors and Markets: Portfolio Choices, Asset Prices and Investment Advice*, Princeton University Press, 2007

# Reverse Optimization

- For each asset, adjust historic asset returns by adding (subtracting) a constant to (from) every historic return so the market portfolio will be optimal for an investor with representative preferences
- If desired, the constants can be modified to reflect an analyst's views about asset mispricing

# Specifying the Representative Investor's Preferences

- If the representative investor's preferences are approximated with a mean/variance utility function, expected utility reverse optimization will give the same results as the Capital Asset Pricing Model
- But, if a different approximation of the representative investor's preferences is utilized, expected utility maximization may provide a more realistic set of possible future asset returns
  - This can lead, in turn, to more realistic predictions of likely future portfolio returns and better asset allocations

# Now and Then

- There are now many more investment vehicles with complex return distributions:
  - Alternative investments
  - Hedge funds
  - Derivatives
- We now know more about the preferences of:
  - Individual investors and institutions, and
  - A representative investor reflecting the preferences of all investors and institutions

# Conclusions

- There is no need to make the restrictive assumptions associated with mean/variance analyses
- Thus for cases in which investor preferences and/or return characteristics make the mean/variance approach inferior, the expected utility approach should be seriously considered as an alternative